The above considerations yield two conclusions: (1) rather mysteriously, retardation is enough to obtain the right potentials, as Liénard and Wiechert did before special relativity, but retardation (as used by Van Flandern) is not enough to obtain the right fields; and (2), the Lorentz transformations predict that an observer with respect to whom two masses are moving will observe a 'magnetic' velocity dependent gravitational force. We now turn to the gravitational equivalent of the Liénard and Wiechert potentials, and present what we believe Eddington had in mind when he declared that the argument given by Van Flandern was fallacious. The only difference between the potential theory of gravitational and electrostatic fields is that the electrostatic potential may have either sign. This being the case, one may compute the Liénard and Wiechert potentials for a particle of mass m moving with velocity $\boldsymbol{v}$. A particularly simple and clear derivation in the case of electromagnetics is given by Reitz and Milford [4], and a clear discussion and concise exposition by Panofsky and Phillips [5]. The formulas obtained are, of course, identical to those obtained by applying a Lorentz transformation to the fields of a static charge. In the gravitational case, one obtains for the potentials

$$\phi = G\frac{m}{s}, \quad A = \frac{1}{c^2}\phi \boldsymbol{v}, \tag{1}$$

where

$$s = r_0\left[1 - \frac{v^2}{c^2}\sin^2\psi\right]^{1/2}, \tag{2}$$

and the geometry is shown in Fig. 2. From the figure, it is clear that

$$\sin\psi = \frac{\left(y_0^2 + z_0^2\right)^{1/2}}{r_0}, \tag{3}$$

so that s may be written as

$$s = \left[x_0^2 + Y_0^2 - \frac{v^2}{c^2}\left(y_0^2 + z_0^2\right)\right]^{1/2}. \tag{4}$$

We have retained c for the velocity of propagation but additional discussion, given below, is needed to assert this is actually the velocity of light.

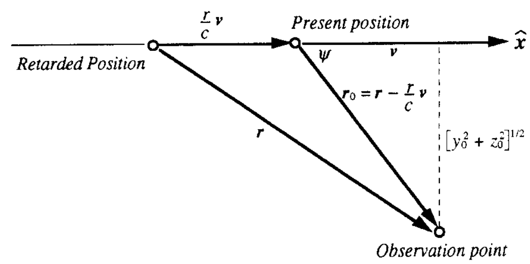

Fig. 2. Field coordinates and geometry for a particle of mass m moving in the x_0-direction with velocity $\boldsymbol{v}$.

The gravitational equivalent of the electric field is then given by

$$\boldsymbol{g} = -\nabla\phi - \frac{\partial \boldsymbol{A}}{\partial t} = -\nabla\phi - \frac{\partial \boldsymbol{A}}{\partial x_0}\boldsymbol{v}. \tag{5}$$

Since the motion is in the x_0-direction, the gradient becomes

$$\frac{\mathrm{d}}{\mathrm{d}x_0} = \frac{\mathrm{d}s}{\mathrm{d}x_0 s}\frac{\mathrm{d}}{\mathrm{d}s} = \frac{x_0}{s}\frac{\mathrm{d}}{\mathrm{d}s}.$$

Straightforward substitution gives

$$\boldsymbol{g} = Gm\frac{\boldsymbol{r}_0}{r_0^3}\frac{1 - \frac{v^2}{c^2}}{\left(1 - \frac{v^2}{c^2}\sin^2\psi\right)^{3/2}}. \tag{6}$$

The key point, particularly with regard to Van Flandern's argument, is that *the the field is directed along $\boldsymbol{r}_0$, the line joining the observation point to the present position, not the retarded position.* We believe this result is what motivated Eddington to claim the argument presented by Van Flandern is fallacious.

There is also a gravitational equivalent of the magnetic field called the 'gravitomagnetic' field in general relativity. And it is the fact that one obtains similar expressions in the weak field 3 + 1 formulation of general relativity [6] that allows one to identify the velocity of propagation, given above as c, with the velocity of light.