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Transformation to Rotating Coordinates, reply to Atwater

ATWATER[1] in his treatment of the transformation to rotating coordinates arbitrarily applied a simple Lorentz transformation to the periphery of a rotating disk having a symmetrical pattern and obtained, by using a constant radius, an asymmetrical pattern. He concluded that the special theory of relativity (STR) may not be applied to the problem of the rotating disk. I think that this conclusion may be unwarranted.

The equivalence principle, which locally equates an accelerated frame of reference with a gravitational field, has not only been a foundation of general relativity but has unfortunately also clouded the division between the domains of applicability of the special theory and the general theory of relativity. The STR is here taken in the sense of Bergmann[2]; that is it is extended to include non-inertial frames of reference which are covariantly characterized by their Riemann curvature tensor being zero. Acceptance of this point of view gives an unambiguous criterion for the applicability of the STR.

In terms of the non-inertial reference system fixed to the rotating disk, the metric may be written as

$$ds^2 = (c^2 - \omega^2 r^2)dt^2 - 2\omega r^2 d\varphi dt - dz^2 - r^2 d\varphi^2 - dr^2 \qquad (1)$$

It is straightforward explicitly to show that the curvature tensor $R^a{}_{bcd} = 0$, as expected. The problem of the rotating disk is then in the domain of the STR, in spite of the non-Euclidean character of its spatial geometry.

It should be noted that rigorously it is not possible to apply a simple Lorentz transformation to the periphery of a rotating disk because the velocities at each point on the periphery are not collinear. (A series of non-collinear Lorentz transformations is of course equivalent to a simple Lorentz transformation plus a rotation, which is the origin of the well known Thomas precession.) Lorentz transformations are representable as rotations in Minkowski space, and the problem of composing non-collinear velocities is equivalent to that of spherical trigonometry on a sphere of radius i. A Lorentz transformation becomes a geodesic arc on the surface of the sphere. A cycle[3] of Lorentz transformations applied to a spatial trihedral at the periphery of the rotating disk does not return the axes of the trihedral to their original orientations, but rotates them through an angle equal to the spherical excess of the geodesic polygon corresponding to the cycle. The trihedral precesses with an angular velocity ω_T, where

$$\omega_T = [(1-\omega^2 r^2/c^2)^{-\frac{1}{2}} - 1]\omega \qquad (2)$$

and this is directed parallel to the angular velocity of the disk.

To ascertain what a material rotating disk actually looks like with respect to an inertial frame of reference we must examine not only these kinematical considerations, but also questions of elasticity and dynamics[4,5].

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