

Non-Homogeneous 3-Space Expansion

Gerald E. Marsh

Introduction

The Friedmann-Lemaître spacetimes thought to represent our universe have exact spherical symmetry about every point, which implies that the spacetime is spatially homogeneous and isotropic admitting a six-parameter group of isometries whose orbits are space-like three surfaces (constant time) of constant curvature (positive, negative, or flat). One may choose the coordinates such that the line element has the form $ds^2 = dt^2 - R^2(t)dl^2$, where dl^2 is the line element of a time-independent Riemannian three-space of constant curvature, be it positive, negative, or flat, and $R(t)$ is the expansion function. What this form of the metric tells us is that the proper physical distance dl between a pair of comoving galaxies scales with time as $l(t) \propto R(t)$. For flat three-dimensional space, now believed to represent the actual universe, the function $R(t)$ monotonically increases with time (The flatness of 3-dimensional space does not necessarily imply that the full spacetime is flat). One can readily show from the form of the metric that the velocity of separation of two comoving galaxies, V , is given by $V = [\dot{R}(t)/R(t)]l$, where the “dot” means the derivative with respect to time. This is the origin of the cosmological red shift. Thus, if $R(t)$ is constant, $V = 0$, and motion freezes.

The parameter t of the Friedmann-Lemaître spacetimes is explicitly identified with the time parameter used to express physical relationships such as in Newton’s and Maxwell’s equations. This implies that if the time is set equal to a constant number so that the universe freezes at some radius, the time associated with physical processes also freezes—nothing can propagate or change in three-dimensional space. Motion and the flow of time are inexplicably linked. This would also be true in more general spacetimes where time may pass at different rates depending on the local mass-energy concentration. Thus, identification of the Friedmann-Lemaître time parameter (often called cosmic time) with physical time implies that the “flow” of time in three-dimensional space is due to the expansion of the universe.

Inhomogeneous 3-space expansion

The best value today for the Hubble Constant is 73.5 km/s/Mpc. $1 \text{ pc} = 3 \times 10^{16} \text{ m} = 3 \times 10^{13} \text{ km}$. So that $1 \text{ Mpc} = 3 \times 10^{22} \text{ m}$. If two points in 3-space are separated by 1m the expansion rate would then be $7.35 \times 10^{-22} \text{ m/s}$. The Hubble Constant appears to be a function only of

distance, but time itself is a function of location in space and nearby matter, so that the Hubble Constant is then a functional (a function of a function).

Consider the space near a Kerr solution to the Einstein field equations. The Kerr metric uses the standard definitions

$$\Sigma = r^2 + a^2 \cos^2 \theta \quad \text{and} \quad \Delta = r^2 - 2mr + a^2,$$

where m is the mass and $a = J/m$ is the angular momentum per unit mass; geometric units are used where $G = c = 1$.

In Boyer-Lindquist coordinates, for an observer stationary with respect to infinity with $dr = d\theta = d\phi = 0$, the derivative of the proper time interval τ with respect to coordinate time dt is given by

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{2mr}{\Sigma}}.$$

Note that $2m$ is the Schwarzschild radius and for $m = 0$, $d\tau/dt = 1$. Because the Hubble "constant" is a functional of both space and time, 3-dimensional space expands more slowly near matter so that the expansion of 3-space is inhomogeneous, contrary to the usual assumption of homogeneity.

A possible analogy with 2-space might be the usual expanding rubber balloon analogy where dots (representing galaxies) on it illustrate the expansion of the universe but where now the thickness of the balloon varies, so that thicker parts expand more slowly than the thinner parts. This generalizes the usual analogy where matter, which is gravitationally or otherwise bound, does not expand with the expansion of 3-space, but 3-space expands more slowly when near matter.