DOES GOD PLAY DICE? CONCEPTUAL QUANTUM PHYSICS FOR THE PERPLEXED

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. . . . .for Adrian

Preface

The late 19th century and the beginning of the 20th brought with it a revolution in the scientific understanding of the universe around us, one whose effects are still being felt around the world as it forces people to change their conception of the universe and the place of human beings within it. We now understand the evolution of the universe from first few moments of its coming into existence some fourteen billion years ago—from the creation of matter to the formation of stars and galaxies. Even a conceptual understanding of the early origin of matter requires an introductory knowledge of quantum mechanics.

Our conception of our own place in the universe also continues to dramatically change. It is now known from direct observation that almost all stars have planetary systems and we can expect to soon find evidence that many harbor life forms. Advances in biology, and our understanding of the evolution of life, have also grown enormously. This new knowledge will enable us, during this century, to design life forms for various practical purposes raising many religious and ethical questions. This has already begun. Our current understanding of the physical world including its biological aspects compared to even half a century ago is simply stunning! It cannot help but inspire a sense of awe and wonder in those who are fortunate enough to come to understand it.

Science up until the beginning of the 20th century could be traced back to its ancient Greek origins beginning perhaps in the sixth century B.C. Its evolution became what is known as the classical view of the world and in particular of physics. The beginning of the 20th century brought with it two great revolutions in physics both due to Albert Einstein. The first was special relativity to be followed later by general relativity or the theory of gravity; the second was quantum mechanics initiated by Einstein's discovery of the photoelectric effect. The attempt to reconcile quantum mechanics with concepts brought over from classical mechanics has led to an enormous literature on the foundations of quantum mechanics and much confusion especially among non-physicists and students of physics. Part of this is due to the historical approach to teaching the subject coupled with the understandable struggle to carry over the basic concepts of particle and wave from classical physics. This essay is an attempt to address some of this wide spread confusion.

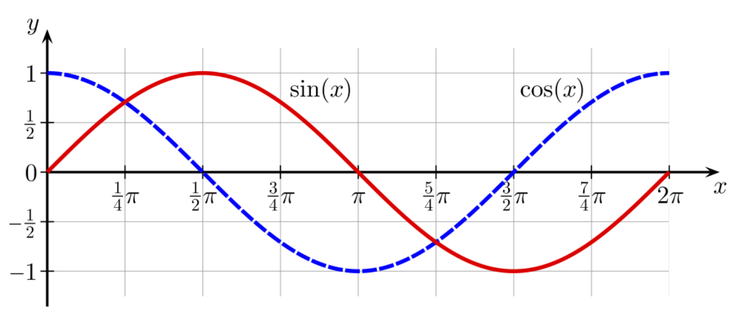
Einstein, in a December 1926 letter to Max Born, speaking of the "secret of the Old One", said that he was "convinced that He does not throw dice". And when Philipp Franck pointed out to Einstein, around 1932, that he was responsible for the idea because of papers he published during his *annus mirabilis* in 1905, responded that "Yes, I may have started it, but I regarded these ideas as temporary. I never thought that others would take them so much more seriously than I did." Later, Einstein put it this way to James Franck: "I can, if the worst comes to the worst, still realize that the Good Lord may have created a world in which there are no natural laws. In short, a chaos. But that there should be statistical laws with definite solutions, *i.e.* laws which compel the Good Lord to throw the dice in each individual case, I find highly disagreeable." `

Einstein was not alone in being uncomfortable with the statistical nature of quantum mechanics and since then a vast literature has appeared on the foundations of quantum mechanics driven at least in part by an attempt to come to terms with its unusual and counterintuitive features. Quantum mechanics is called statistical because having determined the position of a particle in either space or time to within a given small region it is not possible to predict exactly where it will be in the future. Quantum mechanics can only give the probability of its being found in any given region. This raises the question of whether the position of a discrete particle can in principle only be predicted statistically by quantum mechanics or, as will be discussed below, whether the formalism of quantum mechanics applies not to a single particle, but rather to an ensemble of systems. The various approaches to interpreting the mathematics of quantum mechanics are motivated by attempting to somehow maintain our ideas about a classical particle in the context of the quantum world.

The Schrödinger equation describes the wave nature of matter and Schrödinger's approach has its origin in the works of Louis de Broglie to be discussed shortly. The differential equation that describes the motion of a particle is known as Schrödinger's equation and solving it gives the wave function, often designated by the Greek letter ψ (*Psi*), associated with the particle. The value of ψ is a function of the location in space and time chosen to evaluate it.

The statistical interpretation of ψ is due to Max Born who, in his 1954 Nobel prize acceptance speech, ascribed his inspiration for the statistical interpretation to an idea of Einstein's: "He had tried to make the duality of particles—light quanta or photons—and waves comprehensible by interpreting the square of the optical wave amplitudes as probability density for the occurrence of photons. This concept could at once be carried over to the ψ-function: $|\psi|^2$ ought to represent the probability density for electrons (or other particles)". Here, $|\psi|^2$ means the square of the absolute value of ψ, which generally has a complex value. To understand this, one must understand the concept of phase.

The phase of any wave is simply where on the wave one is located, measured from some arbitrary origin. The figure below shows the plot of the periodic sine and cosine functions. The measure of the angle along the x-axis is in radians (there are 2π radians in 360 degrees), after which the figure repeats endlessly. Notice that at $\pi/2$ radians the sine function reaches its maximum while the cosine function has the value zero. The cosine and sine functions are said to be $\pi/2$ radians or 90 degrees *out of phase*.



The solution ψ to the Schrödinger wave equation for a free particle—meaning one that is not being subjected to any force—can be expressed as a superposition of monochromatic (having one frequency) plane waves like the sine and cosine functions shown above. These monochromatic waves have the mathematical form $[\cos\phi + i \sin\phi]$, where ϕ is a phase and i is the square root of -1, making the sum in the square brackets an imaginary number. Because this is the case, if Born's probability interpretation of the wave function is to be a real number one must multiply the wave function ψ by its complex conjugate resulting in $\psi\psi^* = |\psi|^2$, where $*$ means complex conjugate. For example, the complex conjugate of $[\cos\phi + i \sin\phi]$ is $[\cos\phi - i \sin\phi]$ and the product gives $[\cos^2\phi + \sin^2\phi]$, which—from elementary trigonometry—is equal to one, a real number.

The question that remains, of course, is: What does this probability density mean? The experimental situation is that a moving particle such as an electron displays wave properties, but only in a certain sense; and that light, or any electromagnetic radiation, displays a particle nature in that Einstein's photoelectric effect shows that it is composed of "photons". And this is where confusion often begins. The term "photon" is often taken to mean that electromagnetic radiation is composed of individual *particles* called photons. What is true is that the radiation is composed of discrete energy packets whose magnitude is determined by their frequency. Intense radiation has enormous numbers of these packets, while the minimum energy that can be radiated is a single packet of energy $E = h\nu$, where h is Planck's constant[†] and ν the frequency of the wave. It is very important to realize that the wave properties of particles described by the Schrödinger wave function have nothing to do with waves that carry energy such as electromagnetic, acoustic, or water waves.

The problems raised by the concept of the photon are beautifully described by M. Sachs. Henri Bacry quotes him in his book *Localizability and Space in Quantum Physics* in the *Lecture Notes in Physics* series:

> "A very old, yet unresolved problem in physics concerns the basic nature of light . . . Still, logical dichotomy and mathematical inconsistency remain in the usual answers to the question: What, precisely, is light?" [And a few pages later he discusses the conceptual difficulties.] ". . . a single photon, which, by definition, has a precise energy, is described mathematically in terms of a plane wave—a function that has an equally weighted value at all points in space at any given time. With this description, then, one would have to say that the single photon is everywhere, rather than somewhere—although it can be annihilated somewhere by looking for it at that particular place! Along with this spatial description of the single photon, it is specified to be continually traveling at the speed of light.

[†] In 1900 Max Planck introduced the idea that the emission and absorption of radiation by matter takes place in finite quanta of energy, while Einstein in 1905 maintained that this was an inherent property of radiation itself.

> To the (perhaps naïve) inquirer, the logical difficulty appears in trying to answer the question: if the photon is everywhere at the same time, and is traveling continually on its own at the speed of light, where is it going to?"

The concept of a wave being associated with the motion of elementary particles was first introduced by Louis de Broglie in his 1924 publication "*Recherches sur la Théorie des Quanta*". The hypothesis that matter as well as light have a wave-particle duality, and that this is a general property of microscopic particles originates with him. What we call the wave function was called by de Broglie an "*onde de phase*" or a *phase wave*. It is a consequence of the relation $E = h\nu$. He also states that "*qu'il ne saurait être question d'une onde transportant de l'énenergie*" (it cannot be a question of a wave transporting energy).

The wavelength of the de Broglie phase wave is given by the formula $\lambda = h/p$, where λ is the wavelength, p is the particle's momentum (its mass times its velocity) and h is again Planck's constant. Notice that for $p = 0$, the wavelength is infinite, which implies that there is no oscillation and thus no phase wave.[†] What this tells us is that de Broglie's phase wave is related to a particle's motion through space and time. Wave functions describe how particles can travel through space from one moment to the next and this motion is not deterministic as it is in classical physics.

The connection of the phase wave with motion can also be seen by keeping in mind that since material particles have mass, special relativity tells us that we can always choose a frame of reference where the particle is at rest; i.e., we can catch up with a moving massive particle so that it is at rest with respect to us. This means that in one frame of reference the particle has an associated phase wave while in another it does not. This is not the case for a wave carrying energy like electromagnetic radiation. There the velocity of propagation is the velocity of light and special relativity tells us that we cannot catch up with the wave and make it stop.

[†] The discussion here excludes relativistic effects. A relativistic formulation would show that when a particle is stationary, it has a frequency of oscillation associated with it called the *zitterbewegung*, which de Broglie thought of as the inherent frequency of the electron.

One of the best ways to see the effect of the phase wave on a moving "particle" is to consider what is known as the double slit experiment[†], where one directs a beam of electrons so that they strike perpendicular to the plane of an opaque barrier having two neighboring narrow parallel slits cut out of it so that the electrons can pass through. Assume that the beam is such that the electrons arrive one behind the other with enough space between them that they do not interact with each other. Behind the barrier a screen is placed that records the pattern of where the electrons passing through the screen strike. If the electrons were classical particles, we would expect the resulting pattern to be composed of a sum of the patterns due to particles passing through one slit with the other closed and the other with the first slit being closed. This is shown in Figure 1 below.

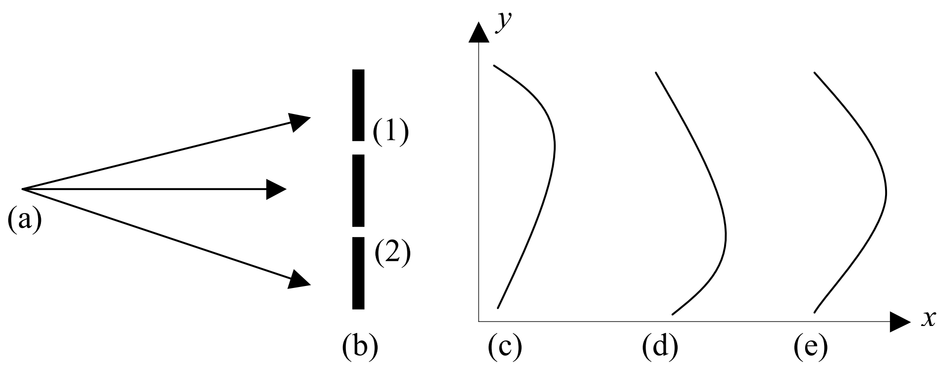


Figure 1. The double slit experiment if electrons were classical particles. (a) is the source of electrons; (b) is the opaque barrier with slits (1) and (2); (c) is the pattern a screen would record when only slit (2) is open; (d) is the pattern the screen would detect when only slit (1) is open. The curve (e) is the detected pattern with both slits being open, which is the sum of (c) and (d). The patterns shown are notional and correspond to the number density (number of electron impacts on the screen per unit area after some time has passed); the *y*-axis is the distance along the barrier and the *x*-axis shows the number density for (c), (d), and (e).

For real quantum mechanical electrons, what one finds for the number density is a pattern corresponding to the interference of a wave that passes through both slits; that is, an *individual* "*particle*" behaves as if it had wave properties and passed through *both* slits—the interference pattern resulting from the interference of the waves coming from the 1st slit with those coming from the 2nd. The intuition that the electron must have passed through only one of the slits must be given up: One cannot determine which slit the

[†] The double slit experiment was originally performed by Thomas Young (1773-1829), an English physician and physicist who is credited with being the founder of the wave theory of light.

"particle" passed through without destroying the interference pattern. A notional idea of the interference pattern is shown bellow.

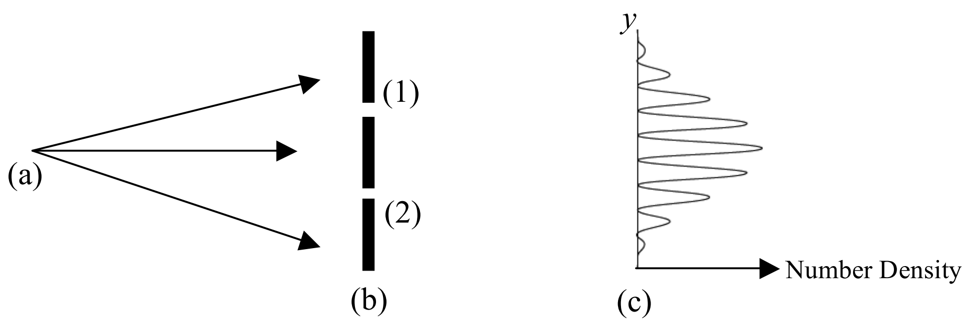


Figure 2. The double slit experiment with real quantum mechanical electrons. (a) is the source of electrons; (b) is the opaque barrier with slits (1) and (2); and (c) is the number density of electrons striking the screen with both slits open. The shape of the curve reflects the interference pattern built up by the wave function associated with individual electrons passing through both slits.

The interference pattern is due to the de Broglie phase wave associated with one "particle", passing through both slits. The number density pattern built up from many individual electrons passing through the slits is proportional to $|\psi|^2$ the square magnitude of the wave function. Note that there are locations on the y-axis where the interference pattern vanishes. These "zeros" of the interference pattern tell us that an electron will *never* strike the screen in those locations—not at all a statistical statement! If this experiment were done with light we would get a similar interference pattern, but that is what would be expected since we now think of light as a wave—there were times in history when this was not the case.

The historical gyrations on the meaning of the Schrödinger wave function derive from the experimental fact that the quantum world, as captured in the wave function or other equivalent formulations, cannot be explained in terms of the classical concepts of a particle or wave. In trying to understand the meaning of the wave function, the first question that should be asked is whether it represents a single system or an ensemble of systems; i.e., does the wave function apply to the motion of a single particle or does it represent the relative frequencies resulting from measuring an ensemble of identically prepared systems. If one holds that the first is true, then there is the question of whether the wave function is a complete description of the system, raising the possibility that there may be unknown or hidden variables that could be specified to make the results consistent with the classical world. By now it has been established both theoretically and experimentally that the possibility of hidden variables can be ruled out. In the literature this is known as Bell's theorem. Bell's theorem basically deals with the concept of what is now known as entanglement, where the state of two quantum particles is correlated. This is discussed in more detail below.

The second possibility, suggested and supported by Einstein, is that Born's statistical postulate should be accepted but interpreted so that the wave function applies to an ensemble of systems—an idea that others further developed. Louis de Broglie also introduced another idea where the wave function could be considered as a kind of "pilot wave" that guides an essentially classical particle into regions where the wave function is large. This concept was further developed by David Bohm culminating in his by now classic papers that appeared in 1952. However, the de Broglie-Bohm theory has never been fully accepted by the scientific community.

Ultimately we must accept the fact that an "elementary particle" is not a "particle" in the sense of classical physics; rather it is some form of space-time excitation that can be localized through interactions, and yet—even when not localized, obeys all the relevant conservation rules and retains "particle" properties such as mass, spin, and charge. This conception is a radical departure from the classical physics notion of a particle, which itself derives from our everyday perceptions and experience. But there is another more familiar example of this from quantum mechanics—the photon, whose interpretational problems were introduced earlier. One can do no better than to again quote Henri Bacry:

> "The photon is not localizable! It is not exaggerate [sic] to say that almost every physicist knows this fact but does not care. A position operator is not an important object. The important operators in quantum physics are the energy, the linear and angular momenta. The spectroscopist, whatever is his field (particle, nuclear or atomic), is not concerned with position! The position operator is only for students and, more precisely, only for beginners in quantum mechanics . . . and for people interested in the sex of the angels, this kind of people you find among mathematical physicists, even among the brightest ones as Schrödinger or Wigner . . ."

One does not need to know the details of operator theory to understand the point of this quote!

Even the name "elementary particle" is deceptive; perhaps "elementary excitation", or some such phrase, would pedagogically lead to less confusion. Instead, one is introduced to the concept of the "wave-particle duality". The problem is due to the use of ordinary language in trying to describe the quantum world. Max Born in his 1957 book *Atomic Physics*, put it this way: "The ultimate origin of the difficulty lies in the fact (or philosophical principle) that we are compelled to use the words of common language when we wish to describe a phenomenon, not by logical or mathematical analysis, but by a picture appealing to the imagination. . . . Every process can be interpreted either in terms of corpuscles or in terms of waves, but on the other hand it is beyond our power to produce proof that it is actually corpuscles or waves with which we are dealing, for we cannot simultaneously determine all the other properties which are distinctive of a corpuscle or of a wave, as the case may be." Born's use of the word "interpreted" should be taken to mean what can actually be measured in an experiment. The attempt to interpret quantum phenomena in terms of classical concepts should be eliminated in pedagogy and the dual nature of the excitations of spacetime that correspond to elementary particles be taught from the first introduction of atoms.

The concept of "spin" is also a carry over from classical mechanics to quantum mechanics of the concept of angular momentum like that of a spinning top. But unlike classical mechanics where angular momentum can take continuous values, in quantum mechanics angular momentum is quantized so that, for example, spin angular momentum (the intrinsic angular momentum of a particle) can only take half-integral values (that is, 0, ½, 1, . . . , where these values are in units of $h/2\pi$).

One should not think of spin as the rotation of an elementary particle. As put by Born, "It is to be noted, however, that the idea of a rotating electron, extended in space, possesses merely heuristic value; we must be prepared, on following out these ideas, to encounter difficulties. (For instance, a point at the surface of the electron would have to move with a velocity greater than that of light, if such values as have been determined experimentally for angular momentum and magnetic moment are to agree with those calculated by the classical theory.)" The heuristic value may have existed in the past, but today it is associated with the historical approach to teaching quantum mechanics and may introduce more confusion than insight.

And, in addition, there is the Pauli exclusion principle: While any number of integral spin particles can occupy the same quantum state, only two half-integral spin particles can occupy the same state, and then only if their spin is opposed. Thus, only two electrons can occupy the same state in atoms; this, coupled with the indistinguishability of electrons, is responsible for the existence of atoms and the periodic table of the elements. Put another way, the quantum numbers of two or more particles with half-integral spin cannot be the same.

Think of a single atom. Its nucleus is localized by the continuous interactions of its constituent components mediated by what is known as the strong force, distinguishing it from electromagnetic and other forces. The electrons surrounding it are localized by their interactions with the nucleus and each other, but only partially, up to the appropriate quantum numbers that describe stable atomic states as a function of distance from the nucleus and total angular momentum and its possible projections along the direction of a magnetic field if it is present. One cannot localize electrons to definite positions in their "orbits"—that being yet another classical concept that does not apply to atoms. Two electrons cannot have the same n, l, j, and m quantum numbers.[†]

In general, the motion of a subatomic particle through space should be thought of as a

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[†] In an atom, an *individual* electron may be characterized by four quantum numbers: $n = 1, 2, \ldots$; $l = 0, 1, 2, \ldots n{-}1$; $j = l{-}1/2, l{+}1/2$; $m = -j, -j{+}1, \ldots +j$. n is known as the principal quantum number and is related to the distance from the nucleus; l is the angular momentum around the nucleus (orbital angular momentum); and j is the total angular momentum of a single electron, which combines its orbital angular momentum with its spin angular momentum. The quantum number m exists if a magnetic field is present, and designates the possible projections of j in the direction of the field. The details of the quantum numbers are not important for what follows.

sequential series of localizations along the particle's path due to interactions. It is not possible to define a continuous path in the sense of classical mechanics, only a series of "snapshots". Between localizations due to interactions, an elementary particle does not have a specific location. This is not a matter of our ignorance; it is a fundamental property of quantum mechanics; an "elementary particle" is not a "particle" in the sense of classical physics. One should not think of the particle existing between localizations due to interactions—there is no "classical little ball" being carried along by the de Broglie phase wave! To reiterate again: A particle is a space-time excitation that can only be localized through interactions and which is characterized by its measurable "particle" properties such as mass, spin, and charge. The real mystery here is the nature of space-time itself that allows such excitations to exist and have the properties they do.

There are several formulations of quantum mechanics, the early ones being those of Schrödinger and Heisenberg (wave and matrix mechanics), which were developed in the 1920s, and which have been shown to be equivalent. A third formulation was developed in 1948 by Richard Feynman[†] based on path integrals, which is also equivalent to the other formulations but is intuitively very appealing. It is based on the two-slit experiment described above. There, electrons could take only two paths set by the slits. If we increase the number of opaque barriers and the number of slits in each, with each increase the number of paths between the source of electrons and any point on the screen also increases. In the limit of an infinite number of barriers and slits it is as if there were no barriers and instead an infinite number of paths between the source and any point on the screen. This process is the basis of the path integral approach to quantum mechanics.

Now consider an initial point and a final point some distance away. Let the path of a moving classical particle pass through both points. If the particle has no forces acting on it, its path would be a straight line passing through the two points. If a constant force is acting on the particle, for example gravity, the path would appear to be curved, but we still assume the two points are on its path. In classical mechanics this path is unique.

[†] The history of Feynman's groundbreaking paper is interesting: His original paper, which laid the foundations for the subject, was rejected by the *Physical Review*! It is often said that Einstein's papers would have been rejected by today's peer review process.

The path integral approach to quantum mechanics considers *all* possible paths from the initial point to the final point.

Now we know from the discussion above that a quantum mechanical particle has a wave function associated with it so that each path will also have a phase associated with it that will be different for each since the different paths have different lengths. Also recall from the example of the sine and cosine functions above that the value of their sum depends on where on the *x*-axis their values are added.

Adding up the phases of each path (called the principle of superposition) at the final point and taking the square of its absolute value—remember, the phase is a complex number—gives the probability that the particle will be found at that particular final point. What this tells us is that a quantum mechanical particle in going from the initial to the final point without interactions takes *all* possible paths between these two points!

The phase associated with each of the paths is a complex number of unit magnitude. But what really matters is not the magnitude but how the different paths interfere with each other due to their differing phase. Paths near the classical path tend to interfere constructively (that is, they have small phase differences), while paths away from the classical path will on the average interfere destructively.[†] As a result, if we go to the classical limit by letting Planck's constant go to zero, the only remaining path is the classical one. Thus, the path integral formulation of quantum mechanics properly reduces in the classical limit.

One of the fascinating concepts in quantum mechanics has to do with correlated systems, a phenomenon now known as "entanglement". These are fascinating because they demonstrate the non-local aspects of quantum mechanics, meaning properties that seem to violate relativistic limitations set by the velocity of light. One already has a hint of this in the path integral formulation of quantum mechanics.

[†] To illustrate this, use the graph of the sine and cosine functions above. Slide the cosine function along the x-axis while not changing the sine function. The result of adding the two together will depend on their phase difference, when they match they have no phase difference and simply add constructively.

There is a mathematical function called the Feynman propagator that integrates the phase along all possible paths of a particle going from an initial to a final point. If one were to include *all* possible paths some would be long enough that the velocity associated with a "particle" traveling along the path would have to exceed the velocity of light. Of course, from the above it should be clear that this has no real meaning since in quantum mechanics we have no way of determining which path the non-localized particle is traveling over. The non-locality makes its appearance here because the Feynman propagator does not vanish outside the light cone.[†] It does, however, decrease very rapidly outside the light cone (exponentially on a scale determined by the particle's Compton wavelength, $\lambda = h/mc$, where m is the particles mass and c the velocity of light).

The concept of entanglement is easy to understand from the following notional example: Assume we have a quantum mechanical particle at rest that has an intrinsic spin $S = 0$. Now let this particle decay into two equally massive particles each having spin $S = ½$. Since the original particle was at rest, the momenta of the two decay particles must point in opposite directions so as to conserve momentum. Similarly, since the spin angular momentum of the original particle also vanished, the spin angular momentum of the two decay particles must point in opposite directions so as to cancel. But since no preferred spin direction existed before the decay, which direction the two oppositely directed spins of the decay particles point after the decay is also not specified. This is illustrated in Figure 3 below by the multiple spin directions at the end of the arrows showing the separation of the decay particles. One should not think of the two decay particles with oppositely directed spins as actually having some specific but unknown direction—the situation is similar to that of the particle paths above; the spin ½ particles occupy *all* possible spin directions until an interaction specifies some preferred direction. Thus, if a measurement of the spin direction of one of the particles forces it to choose a specific direction, the far distant particle will *instantly* be forced to assume the oppositely directed

[†] If one reduces three dimensional space to two dimensions and uses the third dimension to represent time, and then plots the path of particles moving at the speed of light from the origin of a coordinate system (two space axes and one perpendicular time axis) in all possible space directions, one gets a cone called the light cone.

spin direction. Since, experimentally, the distances between the decay particles has often been set up to be meters before spin measurements are made, the velocity of propagation relating the measurement of the first particle's spin direction to that of the second particle, when the second particle is forced into the opposite spin direction, far exceeds the velocity of light.

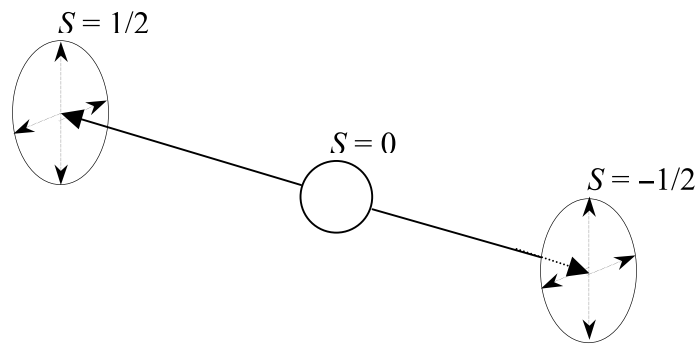

Figure 3. Decay of a spin-zero particle at rest into two equal mass spin ½ particles. Linear momentum is conserved since both decay particles have equal but opposite momenta. The spin ½ particles conserve spin angular momentum because their respective spins point in opposite directions. The actual direction is not specified and the correlated system composed of the two spin ½ particles occupies all directions until one of them suffers an interaction, such as a measurement of spin direction, at which time the other instantaneously assumes the opposite direction.

Many people have argued that this makes quantum mechanics a non-local theory. But there is no violation of special relativity, which forbids only the propagation of energy or information faster than the speed of light. What is being propagated here is a phase relation.

That phase velocities can exceed the velocity of light has been known from classical physics for many years: Groups of electromagnetic or other waves of differing frequencies can be combined to form wave packets that have both a group and phase velocity. The group velocity, which can be used to carry information, must not exceed the velocity of light. When it is slower than the velocity of light, the phase velocity is greater. The form of the relation is $v_g \, v_p = c^2$, where v_g and v_p are respectively the group and phase velocities.

So, the answer to the title of this essay is: Yes, God does play dice, but according to His rules! What we know about the world around us from our own sense impressions, and the formalization of them into the concepts of classical physics, is no guide to the quantum world. It is not what we don't know when entering that world that will deceive and mislead us, but what we do know, and that no longer applies. Quantum mechanics is a self-consistent mathematical theory that describes the microscopic world amazingly well. It is the only guide we have to that world—and what we find there simply cannot be interpreted in terms of classical concepts.