BALL LIGHTNING AS A NON-CONSTANT α SPHERICAL FORCE-FREE MAGNETIC FIELD PLASMOID

Gerald E. Marsh
Argonne National Laboratory (Ret)
gemarsh@uchicago.edu

ABSTRACT

A force-free magnetic field configuration is a state of minimum energy under the constraint of magnetic helicity conservation. It is shown here that force-free magnetic field configurations that can be used for ball lightening can have α in the force-free field equation $\nabla \times B = \alpha B$ a scalar function rather than a simple constant. Such fields have been proposed to contain the plasma of ball lightning.

Tsui [1] has given a comprehensive history of both observations of ball lightning and theoretical attempts to explain it. Many photos and videos of ball lightning are now freely available on the internet. Here is one

Figure 1. Modified photograph of ball lightning in a rural field in the rain. It is the streamer going to ground from the ball that is difficult to explain for most models of ball lightning. [adapted from a free image from Shutterstock].

One puzzling observation is that ball lightning has been seen to go through windows and some walls. One generally associates the electromagnetic field of ball lightning with the atmosphere internal to the ball. Once one realizes that it is not the atmosphere and the electromagnetic field of the ball that goes through walls but only its electromagnetic field configuration, it becomes clear that this is the same as other electromagnetic radiation such as light or radio waves. The difference is that ball lightning has a spherical electromagnetic configuration.

Turning back to the work of Tsui, in his article, he uses the symbol *K* in the force-free field equation rather than the conventional α that will be used here. Force-free magnetic fields are of interest since they constitute a minimum energy state under the constraint of preserving magnetic helicity. That magnetic fields can be used to contain a spherical plasma has been shown long ago [2], [3]. Tsui uses spherical coordinates and decomposes the magnetic field to be used to contain the plasma into poloidal and toroidal components along with the imposition of axial symmetry. Given spherical geometry this means that the poloidal current generates the toroidal field and the toroidal current gives rise to the poloidal field.

Tsui computes the variation of the magnetic energy density under the constraint of conserved magnetic field helicity. He uses α as a Lagrange multiplier. He finds that $\boldsymbol{B} = \boldsymbol{B}_P + \boldsymbol{B}_T$, where *P* and *T* are two scalar functions of (r, θ). The toroidal and poloidal decomposition results in $\boldsymbol{\nabla} \times \boldsymbol{B}_P = \alpha \boldsymbol{B}_T$ and $\boldsymbol{\nabla} \times \boldsymbol{B}_T = \alpha \boldsymbol{B}_P$. This then results in two equations involving two scalar functions *P* and *T* where these functions are coupled; he then decouples them by restricting α to being a constant. Finally, he finds the general solution and as well as the solution outside the radius of the plasma sphere where the magnetic field is described by a magnetic scalar potential Φ that satisfies the Laplace equation. His results are,

$$P(r,\theta) = Cj_l(\alpha r)P_l(\cos(\theta)), \qquad r < R$$

$$\Phi(r,\theta) = Ar^{-l(l+1)}P_l(\cos\theta), \qquad r > R,$$

. (Eqs. 1).

where R is the radius of the plasma sphere.

The restriction of α to a constant value limits the range of spherical force-free magnetic fields that can describe the field within the plasmoid. We use another approach here [4] using a flux function Ψ.

In spherical coordinates (r, θ, ϕ), with the assumption of axial symmetry making the field independent of ϕ, one can introduce a flux function Ψ such that

$$B_r = \frac{1}{r^2\sin\theta}\,\partial_\theta\Psi, \qquad B_\theta = -\frac{1}{r\sin\theta}\,\partial_r\Psi, \qquad B_\phi = \frac{1}{r\sin\theta}\,f(\Psi),$$

(Eqs. 2).

where $f(\Psi)$ is an arbitrary function of Ψ alone.

One may then show that

$$\partial_r^2\Psi + \frac{(1-\mu^2)}{r^2}\,\partial_\mu^2\Psi = -f(\Psi)f'(\Psi),$$

(Eq. 3).

where $\mu = \cos\theta$ and the prime indicates differentiation by Ψ. Also, $\alpha = f'(\Psi)$. Equation 3 is a form of the Grad-Shafranov equation. We now show that there are exact non-constant α solutions to this equation.

One can solve this equation by the methos of separation of variables with a separation constant λ. This results in two equations

$$\Phi''(r) + \left(\alpha^2 - \frac{\lambda}{r^2}\right)\Phi(r) = 0$$

(Eq. 4).

and

$$\Gamma''(\mu) + \frac{\lambda}{(1-\mu^2)}\Gamma(\mu) = 0.$$

(Eq. 5).

If λ is restricted to $\lambda = n(n+1), n = 0, \pm 1, \pm 2, \ldots$ the solutions of Eq. 4 are the Riccati-Bessel functions with argument κr, where κ is a constant; i.e., $(\kappa r)c_n(\kappa r)$ where $c_n(\kappa r)$ is one of $\{j_n(\kappa r), y_n(\kappa r), h_n^{(1)}(\kappa r), h_n^{(2)}(\kappa r)\}$. The properties of these functions follow from those of the spherical Bessel functions. If there is no axial line current, which is what is assumed here, $\kappa = \alpha$.

The solution to Eq. (5) is given in terms of the generalized Jacobi polynomials $P_m^{(\alpha,\beta)}(\mu)$

$$\Gamma(\mu) = (1-\mu)^{(\alpha+1)/2}(1-\mu)^{(\beta+1)/2}P_m^{(\alpha,\beta)}(\mu), \qquad \alpha, \beta = \pm 1, \;\; m = 0,1,2, \ldots$$

(Eq. 6).

provided Eq. (5) is written as

$$\Gamma''(\mu) + G^{(\alpha,\beta)}(m,\mu)\Gamma(\mu) = 0,$$

(Eq. 7).

and $G^{(\alpha,\beta)}(m,\mu)$ is restricted to one of several forms. $G^{(1,1)}(m,\mu)$ will be compatible with $c_n(\kappa r)$ if $m = n - 1$.

Assuming this is the case, apart from an arbitrary multiplicative constant one can obtain for the flux function

$$\Psi = (\alpha r)c_n(\alpha r)(1-\mu)P_n^{(1,-1)}(\mu).$$

(Eq. 8).

Choosing $c_n(\alpha r)$ to be $j_n(\alpha r)$, the magnetic field components are then given by Eqs. (2) as

$$B_r = \frac{\alpha}{r} j_n(\alpha r)\left\{\frac{n+1}{n(1+\mu)}\left[\mu n P_n^{(1,-1)}(\mu) - (n-1)P_n^{(1,-1)}(\mu)\right],\right\}$$

(Eq. 9).

$$B_{\theta =} \frac{\alpha(1-\mu)}{r\, sin\theta} P_n^{(1,-1)}(\mu)\{n j_n(\alpha r) - (\alpha r) j_{n-1}(\alpha r)\},$$

(Eq. 10).

and

$$B_{\emptyset} = \frac{\alpha}{r\, sin\theta}\left\{B^2 + (\alpha r)^2[j_n(\alpha r)]^2(1-\mu)^2\left[P_n^{(1,-1)}(\mu)\right]^2\right\}^{1/2},$$

(Eq. 11).

where *B* in (Eq. 11) is a constant and is zero for no axial current.

In Eqs. (8 to 11), instead of $j_n(\alpha r)$ one could have instead chosen $y_n(\alpha r), h_n^1(\alpha r), or\ h_n^2(\alpha r)$. These equations provide a very large class of axially symmetric solutions that could be used to model the magnetic field in ball lightning.

For *n* = 1, the above gives the following results.

$$\psi = \alpha r\, sin^2\theta\, j_1(\alpha r)$$

(Eq. 12).

$$B_r = \frac{1}{r^2}\csc(\theta)\left((\alpha r(1+\cos\theta)\sin\theta)j_1(\alpha r) - \alpha r(1-\cos\theta)\sin\theta j_1(\alpha r))\right)$$

(Eq. 13).

$$B_\theta = -\frac{1}{r}\csc\theta\left((\alpha\, sin^2\theta)j_1(\alpha r) + \alpha^2 r\, sin^2\theta\ \frac{1}{2}\left(\frac{-j_1(\alpha r)}{\alpha r} + j_0(\alpha r) - j_2(\alpha r)\right)\right),$$

(Eq. 14).

$$B_\emptyset = \frac{1}{r}(\alpha\csc\theta\ \sqrt{\alpha^2 r^2\, sin^4\theta\, j_1(\alpha r)^2}.$$

(Eq. 15).

These vector components are plotted in Figures 2, 3, 4.

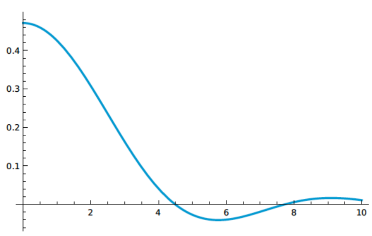

Figure 2. *Br* for $\alpha = 1$ and $\theta = \pi/4$

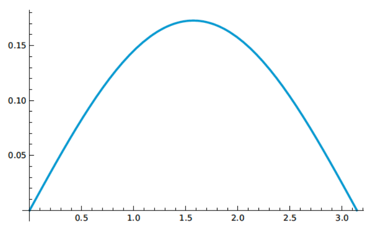

Figure 3. B_θ for $\alpha = 1$ and $r = 5$.

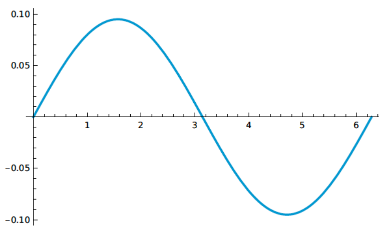

Figure 4. $B\phi$ for $\alpha = 1$ and $r = 5$.

Note from (Eq. 15) that $B\phi$, because of the spherical Bessel function j_1, vanishes for $\theta = \pi$ and from Fig. 4 that it also changes sign for that value of θ. Also, that from Fig. 3, B_θ has its maximum at $\theta = \pi$; in spherical coordinates this is the maximum value of θ.

These vector components provide a rather complex flow in three-dimensional space. This can be seen in Figure 5, where maximum of *r* is set at the first zero of *Br*, which is at $r = 4.5$. This is where boundary conditions are most likely to be imposed for representing the field for ball lightning. The figure is unfortunately difficult to visually interpret in detail and a stream plot of the field does not help in visualization. The overall flow around the ϕ-axis is clearly seen.

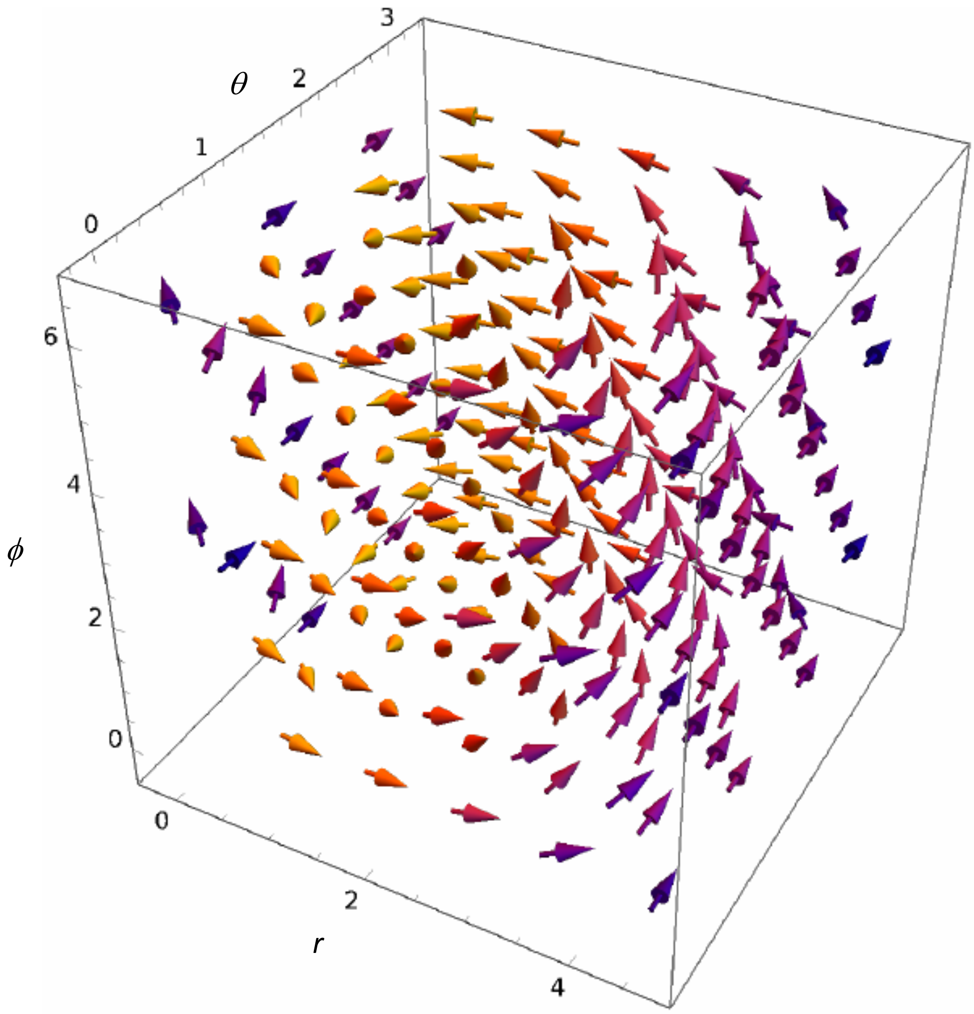

Figure 5. 3-Dimensional vector plot of Br, B_θ and $B\phi$ for a = 1. The range of the variables are {r, 0, 4.5}, {θ, 0, π} and {ϕ, 0, 2π}.

A plot of the flux function can yield some additional insight into Eqs. (8 to 11). With the arbitrary multiplicative constant of (Eq. 8) set equal to unity, Figure 6 shows the n = 1 case for particular values of Ψ as a function of $z = rcos\theta$ and r. The magnetic surfaces outside the central one are toroids with a bannalike cross section.

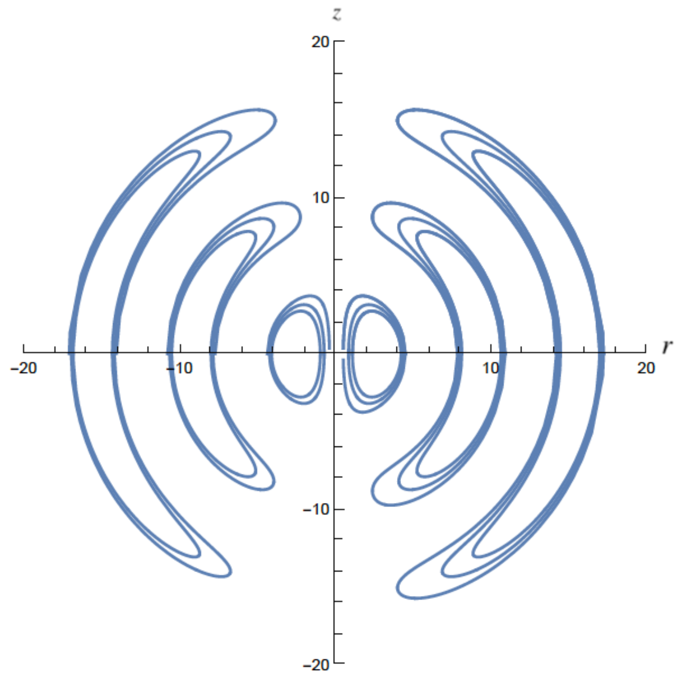

Figure 6. Parametric plot of $\Psi = (\alpha r) j_n(\alpha r)(1-\mu)P_n^{(1,-1)}(\mu)$ for *n* = 1 as a function of $z = rcos\theta$ and *r*. The contours are for $\Psi = \pm 0.1,\ \pm 0.3,\ \pm 0.5$ with that for ± 0.1 being the largest contour. Note that $\Psi = 0$ on the z-axis since it has been assumed that there is no axial line current.

Should it actually turn out to be the case that ball lightning can be faithfully modeled as a plasmoid containing a spherical force-free magnetic field, even the criterion for the choice of which force-free field to use is unknown. In addition, what sort of boundary conditions should be applied at the first zero of *Br* is an open question.

The photographs and videos of ball lightning often show streamers emanating from their basically spherical shape which interact with the local environment. The plasma being contained by the field is neutral, which doesn't mean there cannot be a separation of charge induced by the local environment. The streamers can be thought of as similar to a short lightning stroke. This mandates a short diversion into the nature of lightning.

• • •

On a clear day, the Earth has an electric field of about 100 V/m, with the Earth being negatively charged. This field is induced by lightning strikes over the whole Earth. At a height of ~5 X 10^4 m, the height of the bottom of storm clouds, and the Earth there is a potential difference of ~400 kV. This gives a current of ~10^{-12} Amp/m^2 over the whole Earth. During a thunder storm, the voltage between the bottom of a thunder cloud and the Earth ranges from 20-100 MV; choosing 50 MV as the mean, the electric field between the bottom of the cloud and Earth is ~10^3 V/m. When a lightning strike occurs, negative charges go from the bottom of the storm cloud to Earth.

A lightning strike begins with a step leader that travels from the bottom of the storm cloud toward the Earth for ~50 m where it stops pausing for ~50 μs and then moves in a series of such steps to the ground. The air along the step leader is ionized and this path becomes a conducting path for the negative charges in the cloud. The resulting strong negative charge on the Earth's surface repels the negative charges within the immediate strike zone while attracting large amounts of positive charge. This large positive charge results in the stepped leader inducing streamers up from the ground. When one of these positively charges streamers connects to the negatively charged stepped leader the stepped leader is connected to the ground allowing negative charges from the storm cloud to flow down to the Earth. This leads to an excess of negative charge on the ground which is returned to the cloud in the return lightning stroke. This return stroke is what produces the bright flash of visible lightning and thunder which results from the rapid expansion of the air surrounding the stroke.

• • •

Returning to the issue of the ball lightning streamers, if the plasmoid is assumed to be a perfectly conducting fluid the spherical force-free magnetic field containing the plasma would be frozen into the plasma so that no topological change could occur. A small departure from perfect conductivity at the boundary of the plasma would not only allow the magnetic configuration to change in this region but also plasma to leak out. It is likely that the origin of the streamers is due to this leakage effect. The boundary conditions imposed would likely determine the nature of the leakage of the plasma and its relation to the streamers.

The simple models proposed here for containment by a force-free magnet field do not alone explain these kinds of observations. Nonetheless, they may help in coming to some understanding of the phenomenon.

REFERENCES

[1] K.H. Tsui, *Phys. Plasmas*, **10**, (2003), pp. 4112-4117.
[2] J.A. Angelo, Jr. and D. Buden, *Space Nuclear Power* [Orbit Book Company, Inc. 1985], Chapter 11, pp. 219-221.
[3] S.C. Jardin, *Europhysics News*, **17**, Vol 6 (1986).
[4] G.E. Marsh, *Force-Free Magnetic Fields: Solutions, Topology, and Applications* [World Scientific Publishing Co., 1996].