AN INTRODUCTION TO THE GLASMA HYPOTHESIS

Gerald E. Marsh
Argonne National Laboratory (Ret)
gemarsh@uchicago.edu

ABSTRACT

A brief introduction to the glasma hypothesis is given here. Almost everything covered is far more complex than the discussion given. Nonetheless, this relatively elementary introduction should serve as an entrée into the field. The terminology used in the field is defined and there is an appendix on Light-cone and Milne Coordinates.

INTRODUCTION

The concept of a Color Glass Condensate followed by the formation of a glasma is now used to provide a description for a collision of two relativistic heavy ions and the following evolution of the system. An introduction to these concepts is provided here.

First, a note on terminology: The Color Glass Condensate (CGC) is an extremely dense gluonic state. If $\hat{z}$ is the direction of motion just before the relativistic collision of two hadrons, their gluons can be treated as classical fields which are "static" in the transverse plane where $E \perp B \perp \hat{z}$. After the collision, the field almost instantaneously become longitudinal; i.e., in the $\hat{z}$ direction. Unlike the CGC, the glasma [1], [2] is a rapidly expanding and interacting gluon field. The glasma is the intermediate state between the CGC and the quark-gluon plasma. This state is somewhere between a glass and a plasma hence the term glasma. The lifetime of the glasma is only a fraction of one fm/c, where fm means femtometer and one fm/c is 10^{-23} s.

A description of a collision of two relativistic heavy ions and system's evolution has been given in terms of a spacetime diagram by McLerran and is shown in Figure 1.

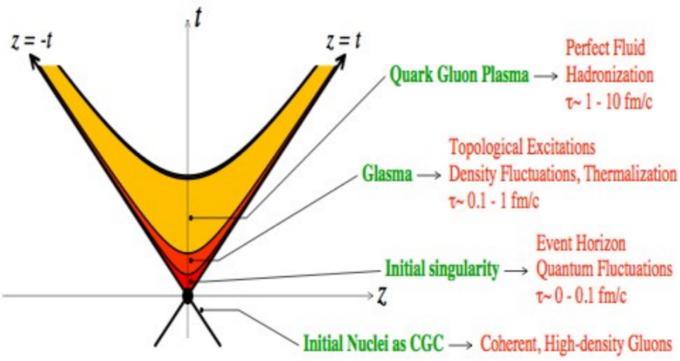


Figure 1. A spacetime diagram for hadronic collisions. McLerran has claimed that this demonstrates the close correspondence between cosmology and the physics of hadronic collisions [L. McLerran, arXiv:0812.4989]

The correspondence of Fig. 1 with cosmology is marginal at best since at the time of the "initial singularity", space-time as we know it did not exist. Thus, the use of Minkowski space is not appropriate for this period of the evolution of the universe. The figure does, however, make perfect sense in the context of the physics of hadronic collisions. Of course, McLerran did not intend that the beginning of the universe could literally be taken to be a CGC followed by a glasma. That would be impossible given the total energy/mass involved.

In what follows, Latin letters *i*, *j*, *k*, . . . correspond to two-component vectors transverse to the longitudinal axis $\hat{z}$ and Latin letters from the beginning of the alphabet *a*, *b*. *c*, . . . for color components of the SU(N_c) color gauge group.

The longitudinal color fields E^z and B^z have been given by Lappi and McLerran [3]as

$$E^z = ig\left[\alpha_1^i, \alpha_2^i\right]$$

$$B^z = ig\epsilon^{ij}\left[\alpha_1^i, \alpha_2^j\right],$$

Eqs.(1)

where α_1 and α_2 are the CGC fields, with all the transverse components vanishing at proper time $\tau = \sqrt{t^2 - z^2} = 0+$.

The field strength is given by 1/g and the Yang-Mills equations (a set of partial differential equations for a connection on a vector or principal bundle) are solved across the forward lightcone with the boundary conditions at $\tau = 0$ being

$$\alpha_3^i|_{\tau=0} = \alpha_1^i + \alpha_2^i$$

$$\alpha|_{\tau=0} = \frac{ig}{2}\left[\alpha_1^i, \alpha_2^i\right]$$

$$\partial_\tau \alpha|_{\tau=0} = \partial_\tau \alpha_3^i|_{\tau=0} = 0.$$

Eqs.(2)

Lappi and McLerran show a numerical solution of the longitudinal color fields given in Eqs.(1). in their Fig. 3. The plot shows that B_z^2 and E_z^2 are related by a function β, that is, $E_z^2 = \beta B_z^2$, where β is only a function of time. The same is true for the transverse fields albeit with a different function. The longitudinal and transverse magnetic and electric color fields are shown in Figure 2.

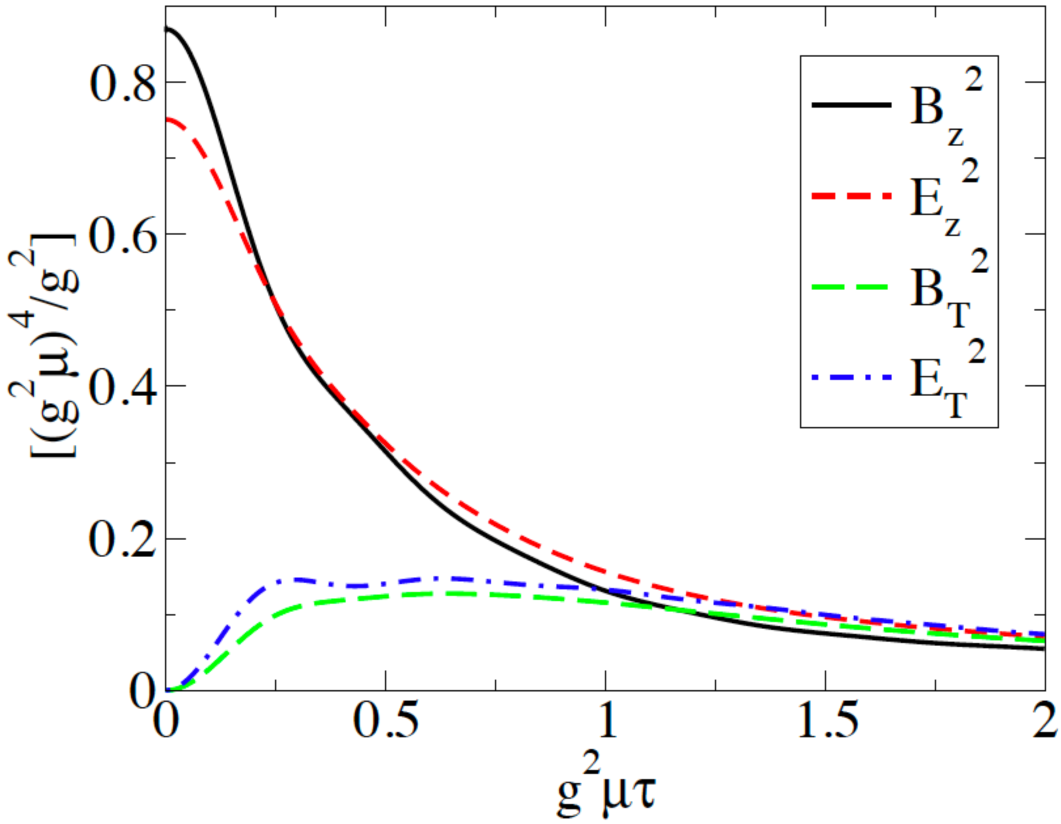
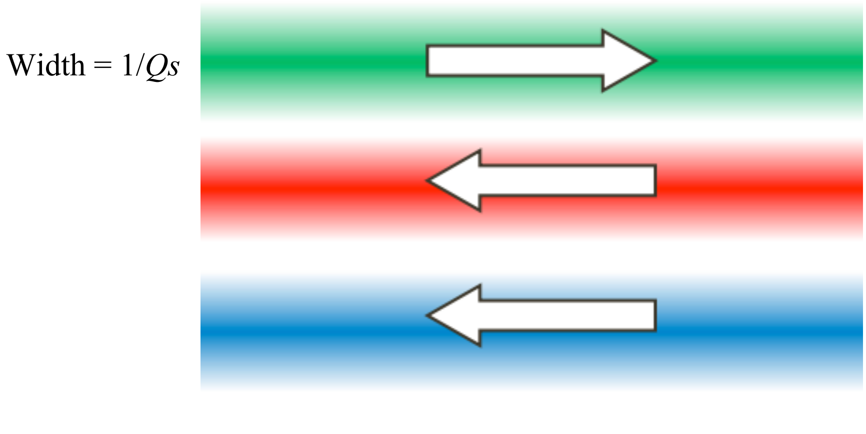


Figure 2. E^z and B^z for Eqs. (1). The figure shows that B_z^2 and E_z^2 are related by a function β. While the function differs only slightly from unity, it is not a constant. The same is true for the transverse fields. The coordinates usually used for the glasma field are the Milne coordinates where the proper time $\tau = \sqrt{t^2 - z^2}$ and $\eta = tanh^{-1}(z/t)$. [From Lappi and McLerran [3].]

Figure 2 shows that even as the transverse field increases the condition $E_z^2 = \beta B_z^2$ is maintained so that the field remains the same until eventually the glasma decays into a quark-gluon plasma. The figure also shows that when the proper time coordinate is essentially zero, only E_z and B_z are non-zero, the transverse fields vanishing. Note that in the usual *z*,*t*-coordinates, $E^z \equiv F^{tz}$ and $B^z \equiv F^{xy}$. At proper time τ = 0, the Poynting vector in the glasma is zero since the fields are purely longitudinal, but as τ increases the transverse components grow creating a flow of energy in the form of photons.

As the density of gluons can increase only to the point where it reaches what is called the Saturation Scale Q_s. Q_s acts as a natural maximum that defines the transverse coherence length. This forces the color fields to be confined to transverse maximum diameter of ~$1/Q_s$ These are called "flux tubes" the boundaries of which are diffuse and can change. $1/Qs$ has the value of about 0.04 to 0.2 fm, where a femtometer is 10^{-15} m.

The color electric and magnetic fields are confined to flus tubes in the longitudinal direction; their color and direction are random since the original fields in the Color Glass Condensate were random. The transverse size is the inverse of the saturation momentum Qs. These are often depicted as shown in Fig.3.



$$E^z = ig[\alpha_1^i, \alpha_2^i]$$

$$B^z = ig\epsilon^{ij}[\alpha_1^i, \alpha_2^j]$$

Figure 3. Color electric and magnetic flux tubes in the longitudinal direction. Note that their color and direction are random. Initially, only E^z and B^z are non-zero. $1/Qs$ has the value of about 0.04 to 0.2 fm, where a femtometer is 10^{-15} m.

Unlike the flux tubes of the Lund string model, these flux tubes can have not only electric flux but magnetic flux or a mixture of both as well [4]. The length of the glasma is $< 3 \text{ X } 10^{-15}$ *m*, and $1/Qs$ is ~0.04 to 0.2 X 10^{-15} *m*.

For a full discussion of electric color fields and currents in uniform fields and in the Glasma see Tanji [5]. For a spatially uniform color field he finds that the field strength $F^{a}_{\mu\nu}$ is given by

$$F^{a}_{\mu\nu} = F_{\mu\nu}\, n^{a},$$

Eq.(4)

where n^a is a constant vector in color space and *a* designates the color. This field strength is given by the gauge field

$$A^{a}_{\mu}(x) = A_{\mu}(x) n^{a},$$

where $A_{\mu}(x)$ gives $F_{\mu\nu}$ as

$$F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}.$$

Eqs.(5)

This means that the usual Maxwell equations can be often used for a spatially uniform color field when Eq.(4) holds. Note that $A^{a}_{\mu}(x)$ is a solution of the Yang-Mills equation.

Since the relationship between E^z and B^z holds for the general fields given by Eqs.(1) it also holds for the fields given by Eq.(4).

The longitudinal electric field generates a longitudinal EM current. Tanji finds that

$$J^{3}_{EM} \simeq \frac{4\, e}{(2\pi)^3} \sum_{c} e^{-\left(\pi \frac{m^2}{|w_c g|E_0}\right)} \times \,(w_c g |w_c g| E_0^2)\, t).$$

Eq.(6)

Here +g/2 is the effective coupling between a color quark with a charge e and the electric field. The uniform color electric field is given by $F_{03}^a = E_0 n^a$ for $t \geq 0$. To obtain Eq.(6), Tanji assumed a strong electric field with $gE \gg m^2$. The effective coupling depends on the direction of the color vector n^a in a way that is gauge invariant. It is given by $w_c g$. For the exact meaning of w_c, see the discussion of his Eq.(8). The absolute value of the effective coupling is of the order of unity.

Here is his plot of the J_{EM}^3 current:

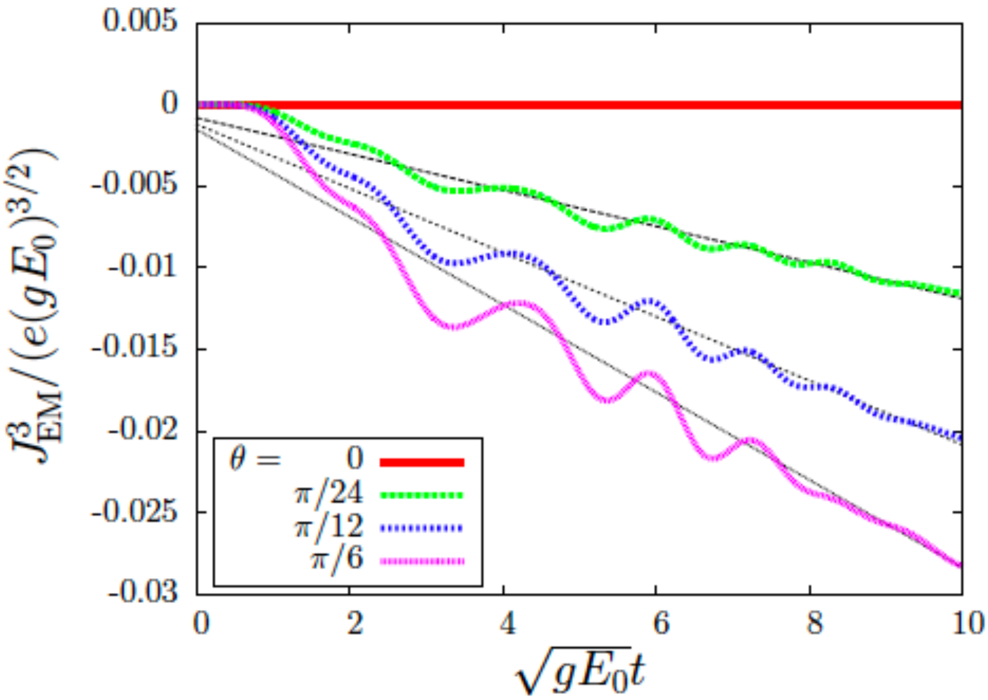


Figure 4. Numerical calculation of the time dependence of the EM current induced by the uniform electric field for various values of the color angle θ for $SU(3)_c$. The thin lines show the approximation given by Eq.(6). [Figure 2 from Tanji [6]].

This limited introduction to the glasma hypothesis for describing the earliest stages after the collision of two relativistic heavy ions addresses the postulate that the very earliest stage is a CGC and that it is almost instantaneously followed by the glasma which decays and thermalizes to form a Quark Gluon Plasma. This plasma then decays to form hadrons. The Glasma Hypothesis was introduced as a way to explore the high-energy limit of QCD.

SUMMARY

A brief introduction to the glasma hypothesis has been given here. Almost everything covered is far more complex than indicated. Nonetheless, this introduction should serve as an entrée into the field. One example of the actual complexity is that if there is an electromagnetic current as in Eq.(6), there will be emission of radiation. It is possible to numerically solve the lattice version of Maxwell's equations due to the $SU(3)_c$ fields of this current [5]. Other issues not covered is that there could also be multiple color fields; and while the Poynting vector associated with the glasma was briefly touched upon, there could, in addition, be a non-zero transverse Poynting vector. See the discussion of this in terms of the McLerran-Venugopalan model by Chen and Fries [6].

APPENDIX

Light-cone and Milne Coordinates

Light-cone and Milne coordinates are used in the context of relativistic collisions of heavy ions to describe the longitudinal expansion of the glasma, the form of matter assumed to exist in the early stage after a collision. The collision axis is chosen to be *z*.

Given the Minkowski space coordinates (*t*, *x*, *y*, *z*), the light-cone coordinates are

$$x^{+} = \frac{1}{\sqrt{2}}(t+z) \quad and \quad x^{-} = \frac{1}{\sqrt{2}}(t-z)$$

Eqs.(A1)

and $x_\perp = (x, y)$ are the transverse coordinates (transverse with respect to *z*). These are shown in Fig. A1.

Milne coordinates are $(\tau, \eta, x_\perp)$. They are curvilinear coordinates used to describe the interior of the forward light-cone. The proper time $\tau = \sqrt{t^2 - z^2}$ are surfaces of hyperboloids in Minkowski space. η is given by

$$\eta = \frac{1}{2}\, ln\left(\frac{t+z}{t-z}\right).$$

Eq.(A2)

In Milne coordinates, the metric is given by $ds^2 = d\tau^2 - \tau^2 d\eta^2 - dx^2 - dy^2$. Some additional terminology: η is known as the "rapidity".

In a collision, τ represents the elapsed time since the moment of the collision for a particle moving at a specific rapidity. In glasma simulations, $\tau = 0$ is when the collision occurs and η is the longitudinal position of the glasma relative to the collision point.

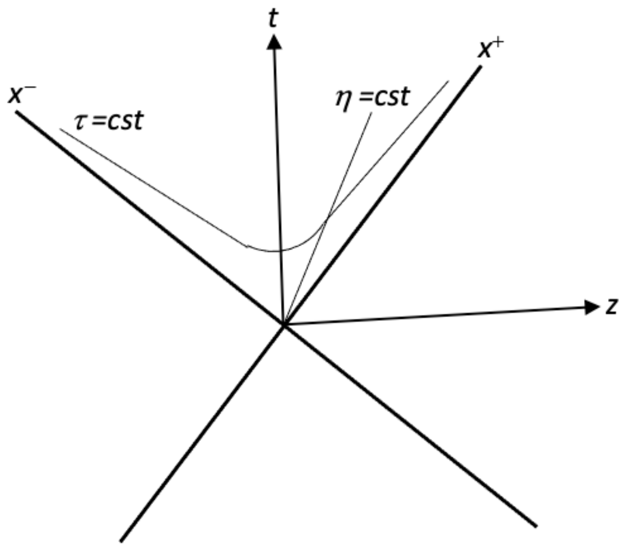


Figure 1. Milne/light-cone coordinates. The surfaces of proper time τ are hyperboloids in Minkowski space. By setting η equal to a constant one can show the linearity of η. Each point in Minkowski space corresponds to a specific value of $(\tau, \eta, x_\perp)$. The transverse coordinates $x_\perp = (x, y)$ are perpendicular to the figure.

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