VOICE OF THE DOLPHINS

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Abstract

Dolphin cognitive capabilities have been explored by investigating their neural anatomy, their social behavior in the wild, and by analysis of their complex vocalizations used for communication and environmental perception. After a brief introduction to dolphin hearing, sounds, and neurophysiology, and an even briefer discussion of sound propagation in the ocean, an analysis is given of some representative vocalizations. It is also shown that Mathematica offers a tool for easily synthesizing dolphin-like sounds that could be as basis for constructing a pidgin type language for human-dolphin communication.

Introduction

The title of this paper comes from the 1961 book *Voice of the Dolphins and Other Stories* by the well-known physicist Leo Szilard written at a time when the world greatly feared the possibility of nuclear Armageddon . . . but perhaps the world could be saved . . . Here is a quote from Szilard's book:

> "In 1960 Dr. John C. Lilly reported that dolphins might have a language of their own, that they were capable of imitating human speech and that the intelligence of the dolphins might be equal to that of humans, or possible even superior to it."

The story then has humanity learning the language of the dolphins and benefiting from their wisdom—but for more, you will have to read the story! The quote from Szilard was based on the fact that Lilly and his colleagues published several papers in the early 1960s showing that dolphins could mimic human speech. Anything more than that was from Szilard's imagination.

This paper is intended to be an introduction into dolphin behavior, hearing, neurophysiology, and the vocalizations dolphins use for echolocation and communication. Most of this is known to researchers specializing in these areas, but not necessarily to those not so specialized. It also introduces some readily available and simple to use tools for the analysis and reproduction of dolphin and other animal sounds. The propagation of sound in the ocean is not simple and cetacean hearing and neural processing must deal with the curved ray paths of sound, multipath, and the non-spectral information contained in echo signals. From such data dolphins may well be able to construct three-dimensional "images" of the objects they echolocate.[1] The "culture" of dolphins has been much discussed and debated,[2] and discussion of this issue is given below.

Dolphins are highly intelligent creatures perfectly adapted to their environment.[†] Consequently, understanding the extent and nature of their intelligence is difficult. One big difference between dolphins and humans is that we have hands, allowing us to adapt to a variety of environments, and our cultural level can be measured by the use of tools and later, after the invention of writing, and

[†] This paper does not even attempt to give a summary of the vast extant literature on cetaceans, the limited references given are those that are directly relate to the subject on hand.

by surviving texts in one form or another. Comparison of dolphin “culture” to that of humans is only reasonable if the comparison is made to the time when humans were still in the hunting and gathering phase, before the invention of agriculture allowed fixed settlements and the rise of civilization, and when they were living in the environment within which they evolved. Even if properly contrasted, given the very different environments it is unclear what measures could be used for comparison.

Like humans, the learning of each generation of dolphins would be lost to succeeding generations were it not for culture as a means of passing down that knowledge. Teaching by example is known to occur in dolphins and perhaps the best-known example is the use of sponges by a small fraction of the population in Shark Bay in western Australia.[3] Foraging and social behavior are taught to calves during their first few years by their mothers. Those mothers that use sponges teach their calves (mostly females) to use conical basket sponges to protect their rostrum when foraging on the bottom. Many other examples of teaching by example exist, but this is the only well-known one where tools are used by wild dolphins. The activity is advantageous to the dolphins because fish found in this way are highly nutritious. Another spontaneous example of tool use is discussed below. If dolphins have a true language there could also be an oral tradition, as was the case with humans up to the time writing was developed.

Is the use of the term “culture” with regard to dolphins justified? As put by Toshio Kasuya, upon receiving an award from the Society for Marine Mammalogy:

> “. . . culture exists if a cetacean species exhibits a behavioral trait that suggests culture, or if it has life history, or social structure that is suitable to maintain a culture, such as the short-finned pilot whales and some other toothed whales.”

Much of the behavior of dolphins and whales is social and comes from being part of a community and therefore could be called cultural—it is very different from the behavior of a school of fish. They can imitate both human and dolphin behavior. An amazing example of this has been given in the book by Whitehead and Rendell. The scientists involved observed the behavior of two female and one male bottlenose dolphin. During the period of observation, a human diver entered the tank the dolphins were being kept in to clean algae from the viewing ports. One can only do justice to what was observed by directly quoting these authors:

> One of the dolphins "was then observed using a seagull feather he had recovered to stroke the viewing glass, apparently copying the cleaning (needless to say, this kind of housekeeping has never been observed in wild dolphins). He then apparently became quite taken with this. He went on to use a variety of objects to fulfill his role as everyone's favorite tank mate, including stones, paper, and even the fish he had been given to eat. He remained quite faithful to his human 'demonstrator' though, even mimicking the human divers' technique of holding onto steel bars beside the window to steady themselves, by placing a flipper in the same spot. He also became quite possessive of the viewing port and would aggressively prevent divers, and the other dolphins, from approaching it for a period of over fifty days. During this time though, he kept the window quite clean."

An even more astonishing imitation occurred when the calf of one of the females was observing the observer through the observation port. At the end of this mutual observation period, the human observer blew cigarette smoke at the port:

> The calf then "swam to her mother, from whom she was still nursing, and returned with a mouthful of milk and spat it back toward the window, producing an uncanny replication of a cloud of smoke. This apparently went on to become a regular trick."

The young, nursing dolphin, who no doubt remembered that milk would make a cloud in the water when ceasing to nurse, had to conceive of the idea of using that observation to reproduce the behavior of the human and do it at the port rather than elsewhere in the tank. You blow smoke at me; I blow the equivalent of smoke at you! Dolphins also have a history of cooperating with humans and in particular with fishermen.

Returning to Toshio Kasuya:

> ". . . it is my view that cetaceans must depend upon knowledge accumulated through past experience. Such knowledge is likely to be transmitted, by learning, to other members of the group. This is the culture of the community."

The book by Whitehead and Rendell is one of the few, if not only, generally readable scientific assessment of dolphin and whale social structure and culture.

Dolphin language

Many animals communicate with each other, but the key elements of true language are semantics and syntax. Semantics roughly has to do with meaning and syntax with the rules for arranging words and phrases to produce well-formed sentences. Herman, et al.,[4] used word order as a measure of syntax and found that dolphins are capable of understanding that changes in word order reflect changes in meaning. As put by him, one dolphin:

> "was able to spontaneously understand logical extensions of a syntactic rule and was able to extract a semantically and syntactically correct sequence from a longer anomalous sequence of language gestures given by a human".

Note the use of the word "gestures". Most dolphin communication with humans is based on the use of gestures and the dolphin's love of imitation. But this is very limited and it is suggested here that creating and using a unique set of "whistles" having varying frequency and amplitude might be more appropriate.

The term "whistle" is somewhat inappropriate. Madsen, et al.[5] have shown that dolphins do not actually whistle in the sense that the whistles are produced by vortices stabilized by resonating nasal air volumes, but rather form the fundamental frequency contour of their tonal calls by pneumatically induced tissue vibrations.

Signature whistles have been studied by Janik, et al.,[6] and the use of a unique set of whistles was to some extent done by Herman, Richards, and Wolz.[7] Their study was designed to determine if bottlenose dolphins could understand imperative sentences expressed in an artificial language. One of their dolphins was taught to recognize computer generated sounds. This part of their study addressed only the comprehension of the acoustic signals used and not the dolphin's ability to reproduce them. Their conclusion was that bottlenose dolphins can understand imperative sentences in terms of both syntax and the semantic components of the sentences using either the artificial language based on acoustic sounds or one that was visually-based.

While both dolphins and whales have specific calls that have particular meanings and understand linguistic syntax, if they have a real language it is probably not one that humans could readily learn since their perceptions are so different. The alternative is to follow the approach of designing an artificial one whose sounds should be easy for dolphins to reproduce. Given such a set of sounds that represent objects, adjectives, and actions, a type of pidgin language could be constructed. That Dolphins can learn such a language has been shown by Richards, et al.[8] The latter part of this paper shows how such sounds can be easily produced.

Introduction to the bottlenose dolphin

A bottlenose dolphin can hear sound in the frequency range of ~75Hz to ~150 kHz, compared to humans whose range is ~20 Hz to ~20 kHz. Dolphins are most sensitive to sounds between about 15 kHz and 110 kHz. As a result, the dolphin recordings that are used here and elsewhere in the general literature do not cover the frequency range of the majority of signals produced by dolphins. It should be noted that the higher frequencies have a much higher absorption coefficient in seawater[9] (~18.3 dB/km at 100 kHz compared to 1.8 dB/km at 20 kHz), so that the higher frequencies are used for relatively local purposes such as echo location. It is now known that dolphins use echolocation to determine the shape of objects and that aural sensory information is integrated with visual information so that mental representations of shape are derived from both sources.[10]

Because dolphins are so superbly adapted to the environment within which they live, how to distinguish learned and instinctual behavior is not always apparent. It is also difficult to use anatomical information to determine the relative intelligence of dolphins. The neural connectivity patterns of their brains are very different from ours indicating that human and dolphin brains evolved along alternate paths to achieve the neurological and behavioral complexity exhibited by both. As put by Lammers and Oswald:

> ". . . less quantifiable, but perhaps most challenging, are the hurdles we must overcome related to the ecological, behavioral, and cognitive differences that exist between humans and dolphins. Although we may share certain similarities as a result of being social mammals, humans and dolphins occupy very different ecological niches and live in distinct sensory worlds. Therefore, we have to assume that substantial differences exist in the way humans and dolphins perceive and communicate about their world."[11]

In terms of the number of neurons in their brains, dolphins bracket the human brain; some having somewhat less than humans and one species—the oceanic dolphin known as the long-finned pilot whale—has more than twice as many.[12]

Both dolphins and whales form cultural communities, exhibit self-awareness, and are capable of planning ahead and understand linguistic syntax. All of which indicates that they are capable of abstract thought. One of the key questions is that while acoustic signals produced by dolphins represent a form of communication far more complex than the minimal form of language exhibited by many other animals, is it in some sense comparable to human speech? This paper will explore some techniques of analysis that could be helpful in addressing this question.

Dolphin sounds

Dolphins produce a variety of sounds and their "whistles" have been much studied. Each dolphin has what is known as a "signature whistle" that identifies it to other dolphins. Whistles are usually continuous and have a duration of tens of milliseconds to several seconds. These are narrow band signals whose frequency, usually between 2 kHz and 20 kHz, varies in time. They can be monotonic or have multiple inflections or steps in their contours. They may also have several harmonics that extend into frequencies greater than 20 kHz. Dolphins are also able to discriminate between whistles by their harmonic content.

The literature often designates a whistle whose frequency varies in time as being FM modulated. This should not be confused with the FM modulation used by FM radio, which has a specific meaning, where the FM signal takes the form

$$FM = BesselJ[0,\delta]\ A_c Sin[\omega_c t] + \sum_{n=1}^{\infty} BesselJ[n,\delta]\ A_c\{Sin[(\omega_c + n\omega_m)t] + (-1)^n Sin[(\omega_c - n\omega_m)t]\},$$

where the Mathematica notational format has been used since Mathematica will be one of the tools used in what follows. The spectrum of this type of FM wave consists of the carrier and an infinite number of sidebands whose amplitudes are various order Bessel functions. In this expression, the amplitude A_c of the carrier is assumed to remain constant. ω_c is the frequency of the carrier, and the ratio of the maximum swing of the frequency from ω_c to the modulating frequency ω_m is called the "deviation ratio" given by δ which is also assumed to remain constant. The magnitude of δ determines the significant number of sidebands. It is possible to amplitude modulate an FM signal as given in the above expression by multiplying by a trigonometric or other type of function. Example of this are given in the section which introduces the Mathematica program used to obtain the envelope of a signal. Dolphin signals are generally more complex than this type of modulation.

There are two anatomical sites for sound production in the dolphin that can be controlled separately so that "whistles" and other sounds can be produced simultaneously.[13] Some of these are high pitched clicks—whose frequency can exceed 100 kHz—that are generally used for echolocation, and various other types of pulsed sounds whose meanings are unclear. It should be emphasized that the signature whistles of bottlenose dolphins, with a frequency and harmonic contour that is unique to the individual, is crucial because it allows them to specifically address each other. Without this capability, their language would be limited to group communication signals such as "danger", "fish", "follow", "help", similar to what is exhibited by some primates and birds.

Neurophysiology

Dolphins process auditory information in at least two areas of their brains—one adjacent to the primary visual cortex and one in the temporal lobe.[14] The external part of the ear, the pinnae, and the external auditory canals were lost over evolutionary time. The lower jaw became the primary channel for sound. The tympanic membrane (eardrum) was replaced with a thin and large tympanic bone plate. The cochlea, while maintaining its basic structural form and function has also evolved to become better adapted to underwater existence. In essence, dolphins rely on sound conduction through a special "acoustic fat" channel in the lower jaw to carry sound directly to the bony case of the inner ear. Thus, the tympanic membrane and its path to the middle ear bones are largely bypassed.

In humans, the hair cells and auditory nerve fibers from the cochlea are limited in the frequency they can respectively generate and carry in response to an audio signal, a frequency ~3 kHz being the upper limit. For frequencies above ~3 kHz, the brain relies on the tonotopically organized basilar membrane of the cochlea (see Fig. 4), which provides a spatial separation of higher frequencies. Separate nerve fibers from the different tonotopic regions of the cochlea then carry information to the (secondary) auditory cortex of the brain to form a tonotopic map where different frequencies go to adjacent regions of the cortex. While the details will certainly differ, a similar solution to the frequency limits of nerve fibers (tonotopic brain maps) must exist in dolphins.

Figure 1 uses the auditory brain stem response to determine the low and high frequency limit of hearing in the common dolphin.[15]

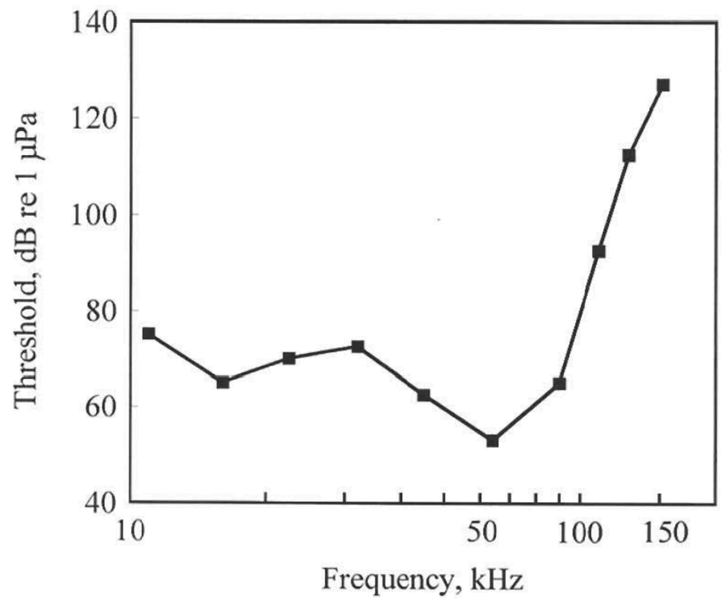


Figure 1. Dolphin auditory brain stem threshold to tone bursts as a function of frequency. [From V.V. Popov and V.O. Klishin, *Aquatic Mammals* **24.1**, 1998].

The unit used to plot threshold response is the decibel, a comparison of sound intensities or energy density even though "dB re 1 μPa" appears to refer to a pressure; more precisely, "dB re 1 μPa" refers to the intensity of a plane wave of pressure equal to 1 μPa.

The common dolphin is smaller than the bottlenose dolphin and has a brain weighing about 800 g compared to 1500 g for the bottlenose dolphin. Humans have brains weighing ~1400 g.

Perhaps surprisingly, the very high frequencies that dolphins hear can also can be heard by humans. SCUBA divers can hear ultrasonic frequencies to greater than 100 kHz, but have no pitch discrimination above ~20 kHz. The evolution of the dolphin auditory system has resulted in greater high-frequency hearing sensitivity and very complex auditory processing.

Individual auditory nerve fibers of the human or dolphin transfer information from only a narrow part of the audible frequency spectrum. Electrophysiological recordings of the threshold response of the nerve fibers to sound are known as tuning curves. They are plotted as the threshold intensity in dB required to achieve a response above the spontaneous firing rate of the associated neurons (in humans from essentially zero to 120 spikes/sec) as a function of frequency. Tuning curves for the dolphin have been obtained by monitoring their auditory brain stem response, and are similar in shape to those seen in other mammalian species. An example of a tuning curve is shown below.

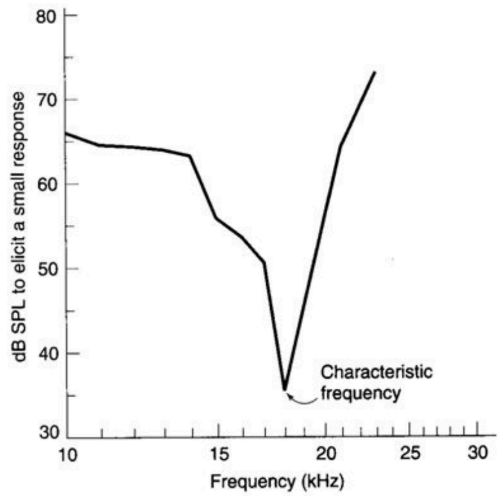


Figure 2. An example of a tuning curve. SPL means Sound Pressure Level and the terminology dB SPL is generally defined for measures of human hearing as $20\ log_{10}\ p_1/p_0$ where p_0 is the reference value in μPa. (Figure from Wikimedia open). The values on the ordinate are often inverted and shown in negative values of dB. It then means that p_0 is greater than p_1.

The method used to obtain such tuning curves is often the "masked threshold" technique: The absolute threshold of sound is the minimum detectable level of that sound when heard alone. The masked threshold is the quietest level of the signal that can be heard when combined with a masking signal. The method consists in presenting a listener with a target tone of fixed level and frequency and measuring the power that a second tone must have to mask the target as a function of the frequency of the masking tone. An example of tuning curves for the common dolphin are shown in Fig. 3.

The "quality" of tuning curves is often given in terms or Q_{10}, the center frequency divided by the bandwidth at a level of 10 dB above the bottom tip of the curve. With few exceptions, the quality is almost constant across a wide frequency range so that in humans an octave occupies a constant interval ~4 mm along the basilar membrane of the cochlea. Thus, in mammals in general, the auditory system can be thought of as a set of frequency tuned bandpass filters where Q_{10} is essentially constant across the auditory frequency range. Known exceptions include one species of bat and the small porpoises classed as *Phocoenidae*.[16] The Q_{10} value for the bottlenose dolphin is about twice as large as that for the common dolphin. Humans have a Q_{10} value close to that of the common dolphin.

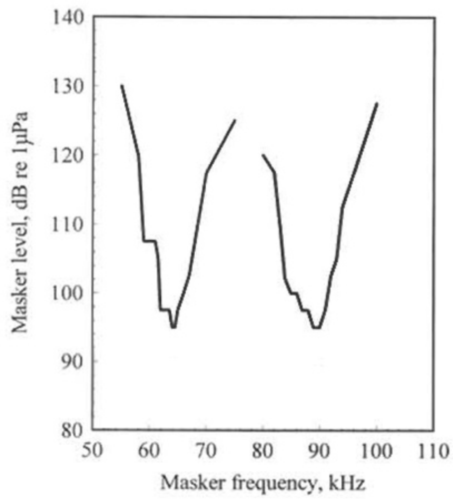


Figure 3. Tuning curves for the common dolphin at the probe frequencies of 64 kHz and 90 kHz. [From V.V. Popov and V.O. Klishin, "EEG study of hearing in the common dolphin, *Delphinus delphis", Aquatic Mammals* 24.1, (1998), pp. 13-20.]

Tuning curves are associated with different regions of the basilar membrane of the cochlea as shown in Fig. 4. Although the figure is for the human cochlea, a similar layout would exist in the dolphin cochlea albeit at higher frequencies.

The vestibulocochlear nerve, known as the eighth cranial nerve, transmits sound and equilibrium information from the inner ear to the brain. The dolphin auditory nerve has several times as many fibers as the human eighth nerve, and the fiber diameters are also about twice as large as in humans, which about doubles their speed of signal propagation. The auditory tonotopic map in the dolphin brain has been displaced from the temporal to the parietal lobe (above the temporal lobe and behind the frontal lobe) and dorsal part of the hemisphere.[17]

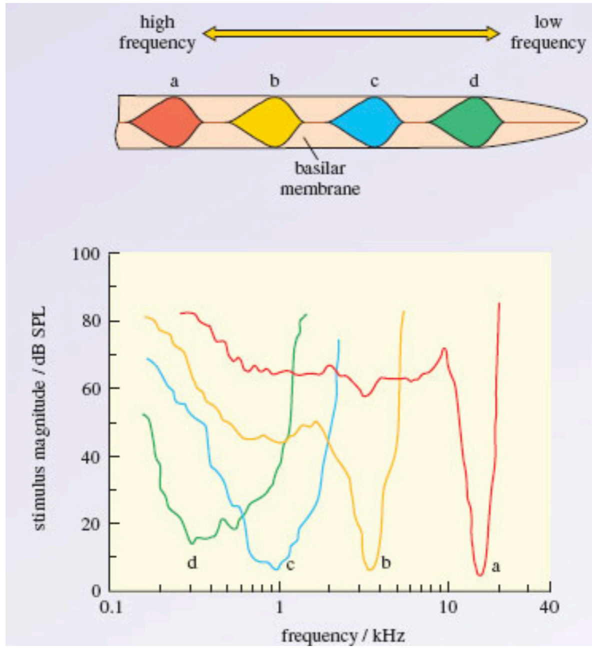


Figure 4. Color coded tuning curves and associated areas of the basilar membrane of the human cochlea. The basilar membrane of the cochlea is shown unrolled. [Figure from Wikimedia open.]

In humans, as pointed out by Vanthornhout, et al.[18] and R.V. Shannon, et al.[19], the primary factor in speech intelligibility is the temporal envelope having a modulation frequency below 20 Hz. The most important frequency range is 4-8 Hz corresponding to the average frequency of speech.

The history of cochlear implants for hearing impaired people illustrates the importance of the temporal envelope. Early models of such implants had only one channel so that they could only provide a single time varying waveform. Nonetheless, recipients of such implants could understand speech; frequency is not the primary factor in speech comprehension.

Figure 5 shows the upper and lower envelopes of an example of human speech. Note the near mirror symmetry of the two signals. The Fourier transform used to produce the spectrum shown in Fig. 6 combines the two signals. These figures were produced during the course of a hearing experiment that used electric currents corresponding to the envelopes to induce cortical entrainment intended to enhance speech comprehension.

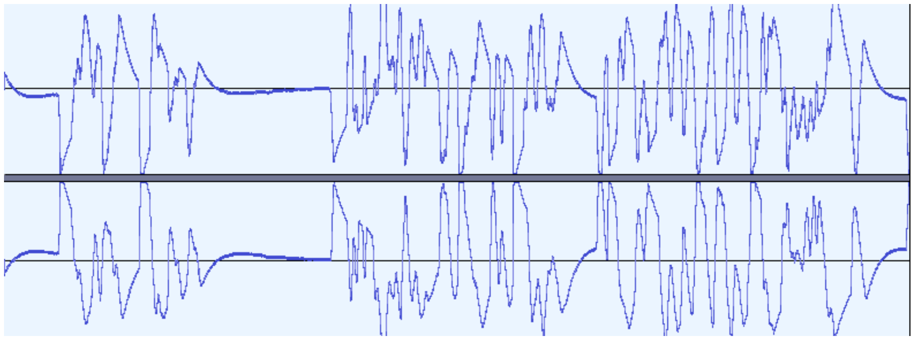


Figure 5. The upper and lower temporal envelopes of a sample of human speech. The original auditory signals were filtered with a second order lowpass Butterworth filter with a corner frequency ~ 13 Hz in order to obtain the envelopes, which should be noted are not quite symmetric. [G.E. Marsh, unpublished]

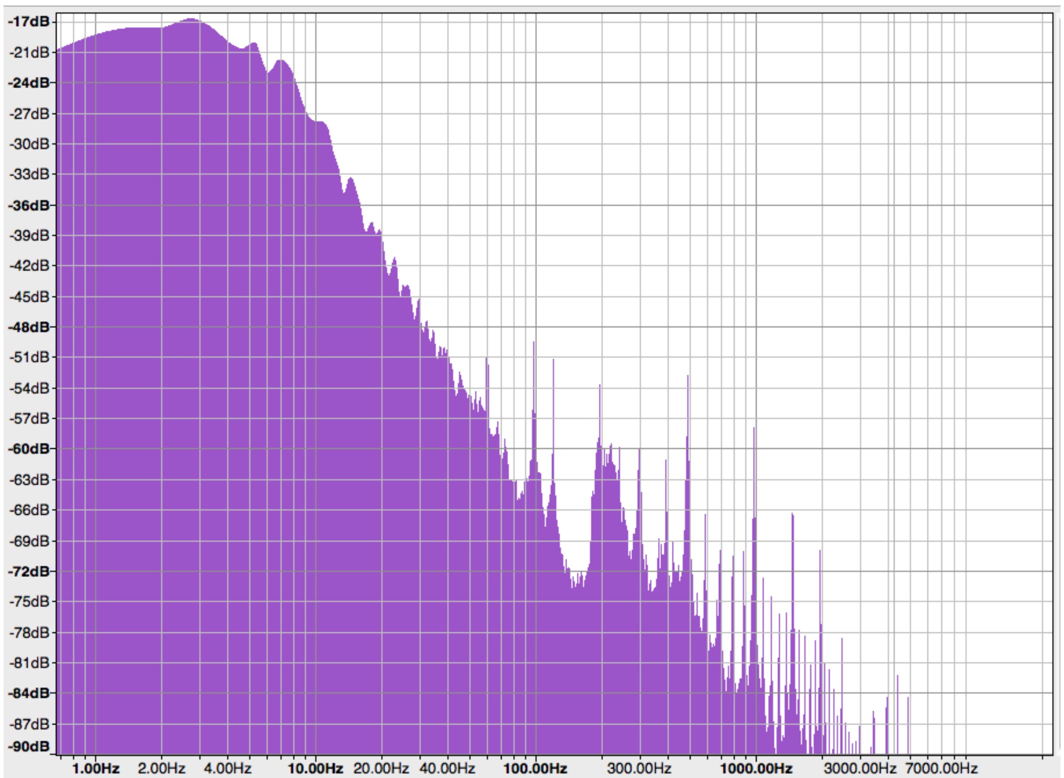


Figure 6. The spectrum of the combined envelopes shown in Fig. 7. [G.E. Marsh, unpublished]

The auditory cortex of non-human primates is composed of areas known as the core, belt, and parabelt regions. In humans, the primary auditory cortex corresponds to the core regions. Kubanek, et al.[20] found that the human non-primary auditory cortex faithfully tracks the speech envelope, a phenomenon often called cortical entrainment or phase-locking. This is shown in Fig. 7.

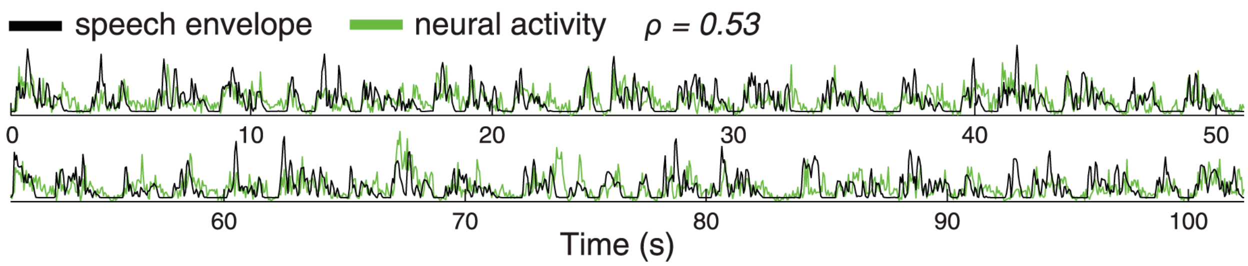


Figure 7. Gamma wave tracking in the human auditory cortex. The neural signal has been scaled to the magnitude of the envelope. ρ is the Spearman correlation coefficient. Gamma waves in the brain have a frequency range of about 30-80 Hz, while high-gamma waves have the range 80-200 Hz. [From J. Kubanek, et al. PLOS ONE **8**, e53398]

Experiments with bottlenose dolphins show that 14 ms tone bursts, amplitude modulated at 600 Hz, evoked a strong auditory neural response that is phase-locked with the envelope of the sound. The modulation index used in the experiment was unity (100% modulation), meaning that the amplitude of the carrier wave was the same as that of the modulating wave. The neural response was monitored as is done when recording an EEG: A recording suction cup electrode was placed on the dolphin's skin about 6 cm behind the blowhole with a ground electrode behind that followed by a reference electrode.[21] That is, the "recording electrode" and the "reference electrode" go to the inputs of a differential amplifier so that in-phase inputs cancel and out-of-phase inputs are amplified with respect to the grounded electrode. This allows signal amplitudes in the microvolt range to be recorded. See also the article by W.F. Dolphin, *Electrophysiological Measures of Auditory Processing in Odontocetes* (Ref. 8, Fig. 7.9 on p. 310).

Figure 8 shows the auditory brain stem response to a "click".

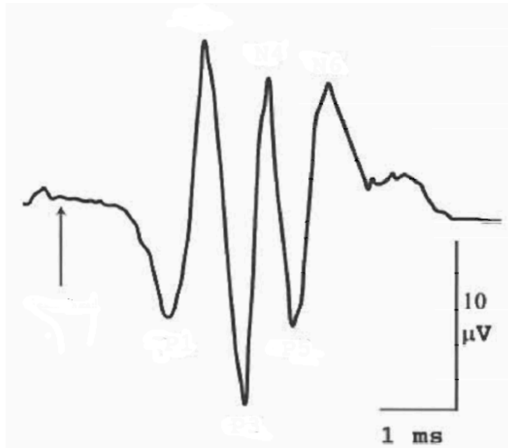


Figure 8. Auditory brain stem response to a 120 dB re 1 μPa click produced by a piezoceramic transducer activated by 5 μsec long rectangular pulses (the arrow corresponds to when the signal reached the dolphin's head where the response signal was recorded). The latency can be seen to be 0.75 ms. Recordings here and in Fig. 9 were from a common male dolphin 1.54 m long (an adult reaches 1.9-2.5 m) that was ill and died after four days of monitoring. [Adapted from V.V. Popov and V.O. Klishin, *Aquatic Mammals* **24.1** (1998)].]

A 0.75 ms latency period corresponds to the ability of the auditory nerve fibers to carry signals up to at least 1300 Hz. The paper by Popov and Klishin contains another figure that is very important for interpreting dolphin signals.

When measuring the response to paired clicks having a separation of 2 ms (corresponding to 500 Hz if part of a continuous series) the responses just merge, as can be seen in Fig. 9. Thus, 300 Hz is likely to be the maximum frequency for communication purposes. There is a peak around this frequency in the spectra shown later in this paper.

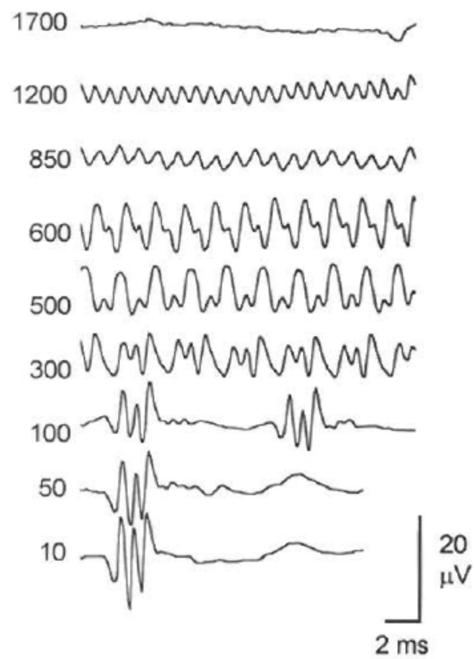


Figure 9. Auditory brain stem response to rhythmic clicks of different rates (indicated by number per second to the left of the traces). The sound intensity was 120 dB re 1 μPa. Note that the response smooths above 200 s^{-1} (or 200 Hz) to form a sinusoidal response at 300 Hz. [From V.V. Popov and V.O. Klishin, *Aquatic Mammals* **24.1** (1998)]

As mentioned above, dolphins also have an envelope following response as can be seen in Fig. 10. The fact that the modulation following response appears up to about 1250 Hz in dolphins does not imply that the maximum frequency used for communication is greater than the ~300 Hz implied by the measurements shown in Fig. 9.

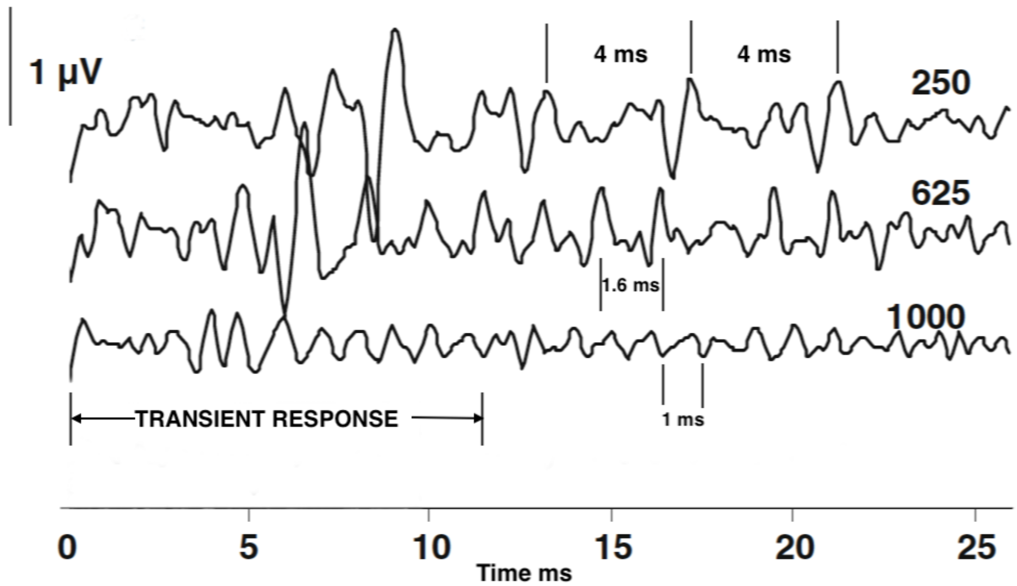


Figure 10. The rate following response of a white-beaked dolphin to click stimuli. The numbers on the right-hand side correspond to the modulation frequency in Hz; the click time interval corresponding to 250 Hz is 1 click every 4 ms, 625 Hz to 1.6ms, and 1 kHz to 1 ms. Responses are an average of 1000 recordings. The entrainment begins after a transient response. (Adapted from T.A. Mooney, et al., J. Comp. Physiol. A (2009) 195:375-384)

A Fourier analysis of the 250 Hz and 625 Hz response traces shown in Fig. 10 clearly show harmonics up to ~1250 Hz.

Dolphins can detect audio signals with a very fine time resolution, their integration times for a single short signal being on the order of 200-300 μs. Integration time is a measure of the ability to isolate individual acoustic events. A longer integration time corresponds to a reduction in temporal resolution. This fine time resolution allows dolphins to use high frequency broadband clicks of microsecond duration to visualize objects around them in terms of distance, shape, orientation, and composition. This is because the echo signal is amplitude modulated and several milliseconds in duration containing a great deal of non-spectral information. With reference to Fig. 10, note that 200-300 μs taken as a frequency corresponds to 500-330 Hz.

The ocean is a complex medium where temperature, density, sound velocity, and salinity vary with depth. There are many factors that affect the return echo from a dolphin's clicks. Sound does not travel in a straight line in the ocean. For example, the mixed isothermal layer, which extends to about 60 m, forms a sound channel where sound emitted at a shallow angle returns to the surface where it is again reflected—see Fig. 11; the return signal from moving object would also be doppler shifted. Over short distances the curved ray paths would not matter, but for communication over longer distances it would, for example for a female dolphin locating a lost calf who is sending a distress signal.

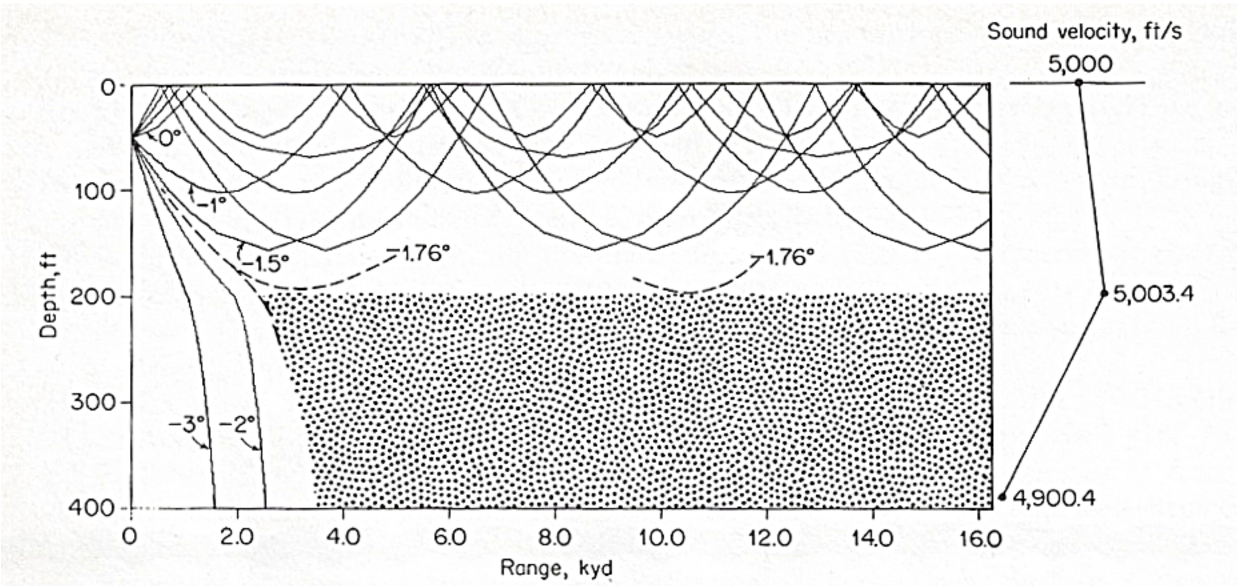


Figure 11. Ray diagram for the transmission of sound from a 50-ft source in a 200-ft mixed layer. [From R.J. Urick, "Principles of underwater sound" (McGraw-Hill Book Co., New York 1983, 3rd edition), Fig. 6.1 (with minor corrections)]

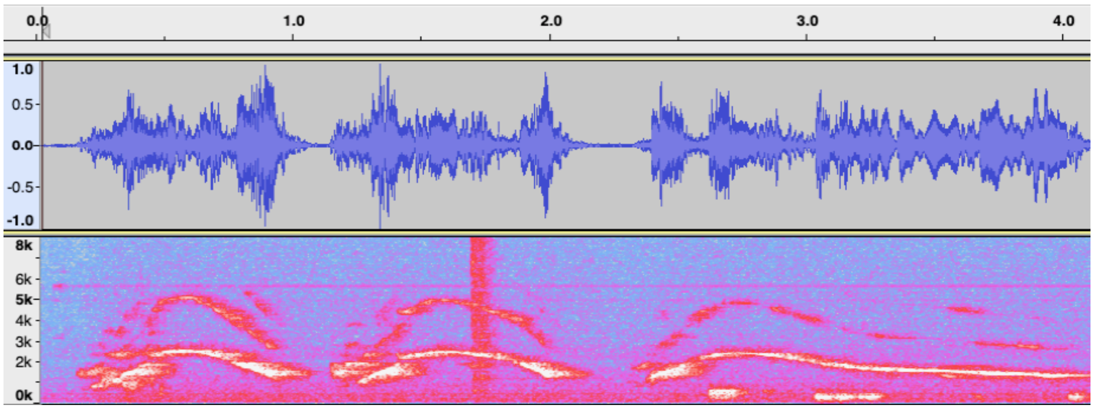
Multipath, where there is more than one propagation path for sound between the dolphin and the sound scattering object, is an important factor in spreading out the return signal. In shallow water, the dolphin must also take into account scattering from the seabed.

Dolphin and human recordings

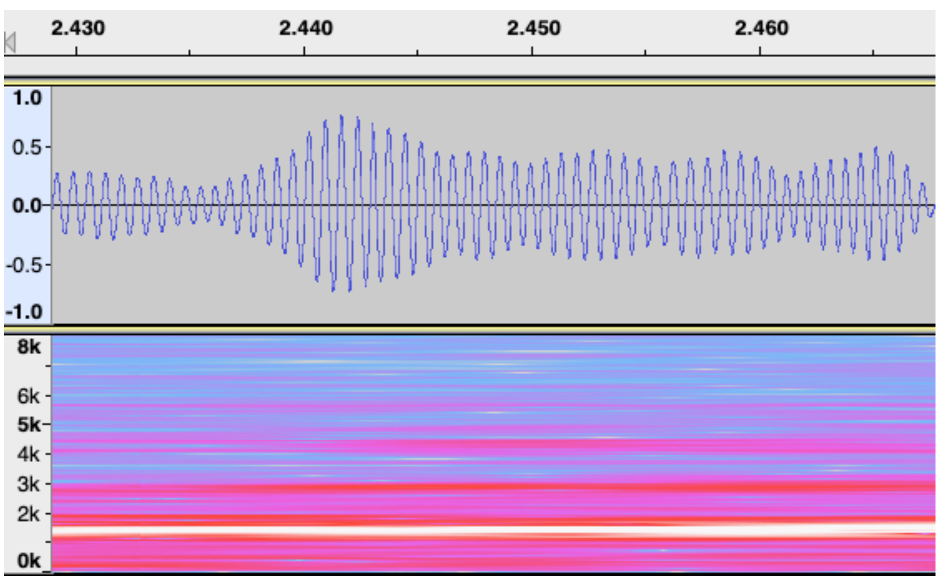
The dolphin recordings that will be used as examples are shown below along with their spectrograms and spectrums.[22] They were recorded by the freely available program Audacity. All the spectra use a Hanning window. Click on the links to hear the sounds.

Dolphin recordings

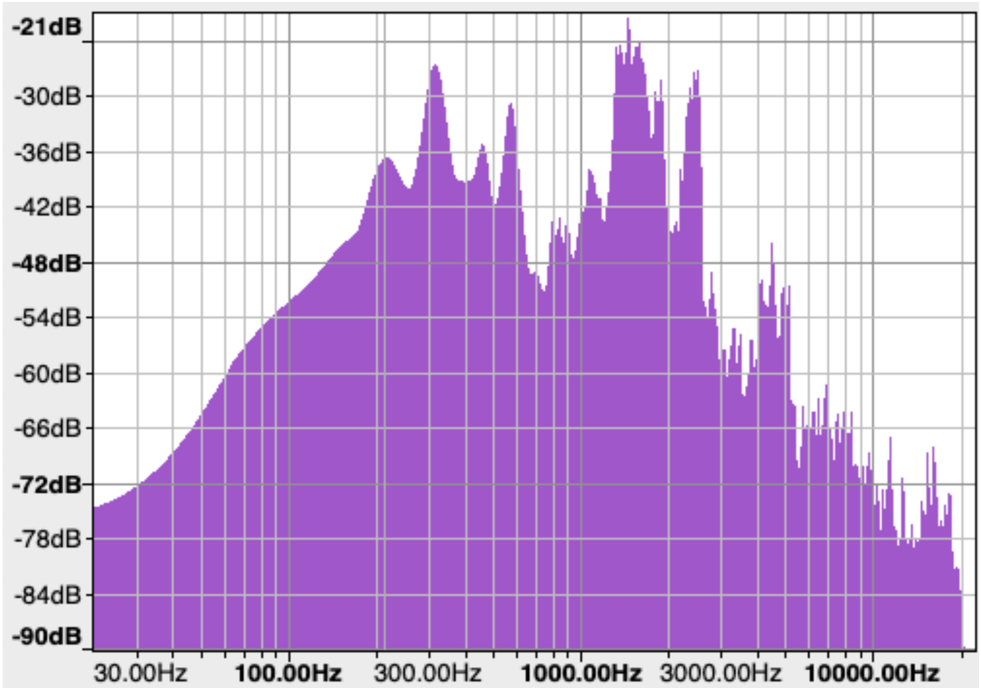
Dolphin 1. Link to Dolphin 1 sounds: http://www.gemarsh.com/wp-content/uploads/Dolph1.wav



Dolphin 1 signal and spectrogram. The signal is listed as being 5.6 sec long (the main portion of the signal is closer to 4 sec). Note the harmonics in red.



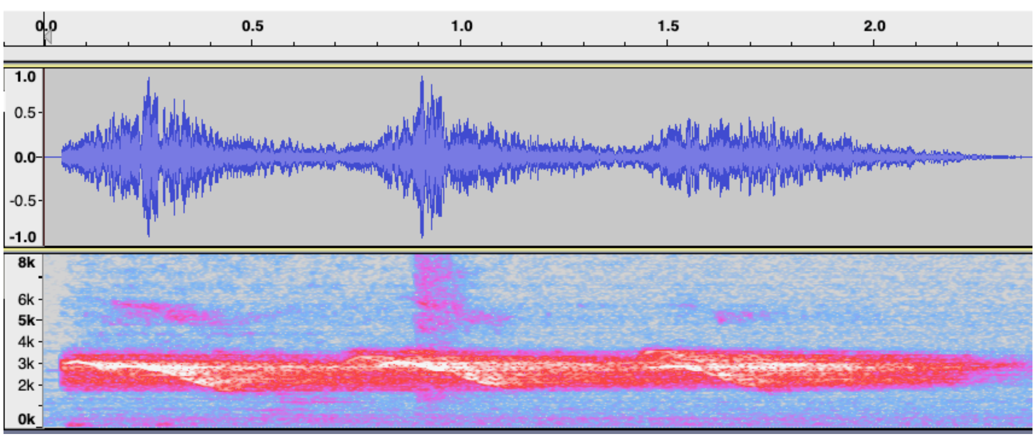
Magnified Dolphin 1 signal. The frequency corresponding to the interval from 0.445 s to 0.455 s is 100 Hz; that between 2.465 s and 2.458 s is 143 Hz.



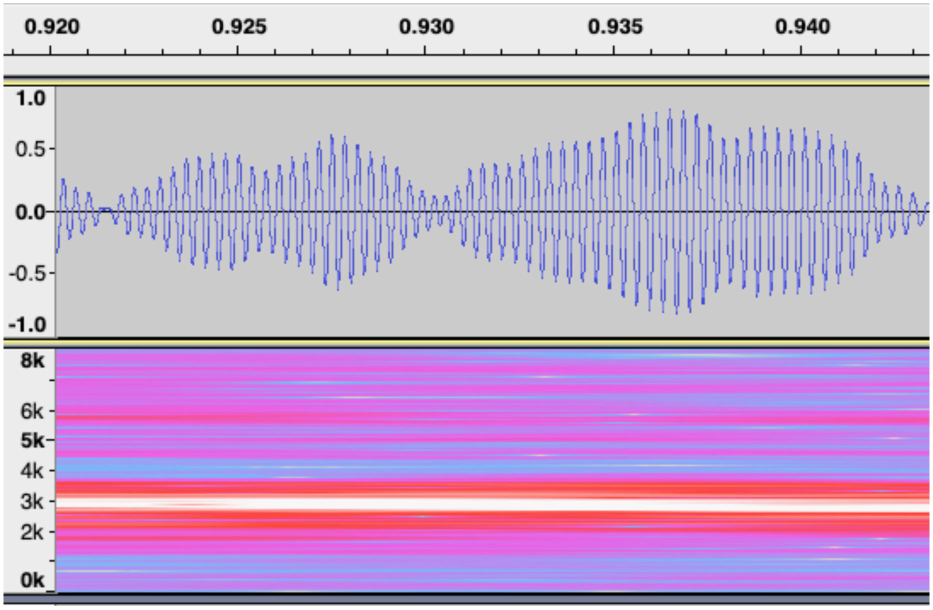
Dolphin 1 Spectrum.

Note that in Audacity dB usually means dB re FS where FS means "full scale".

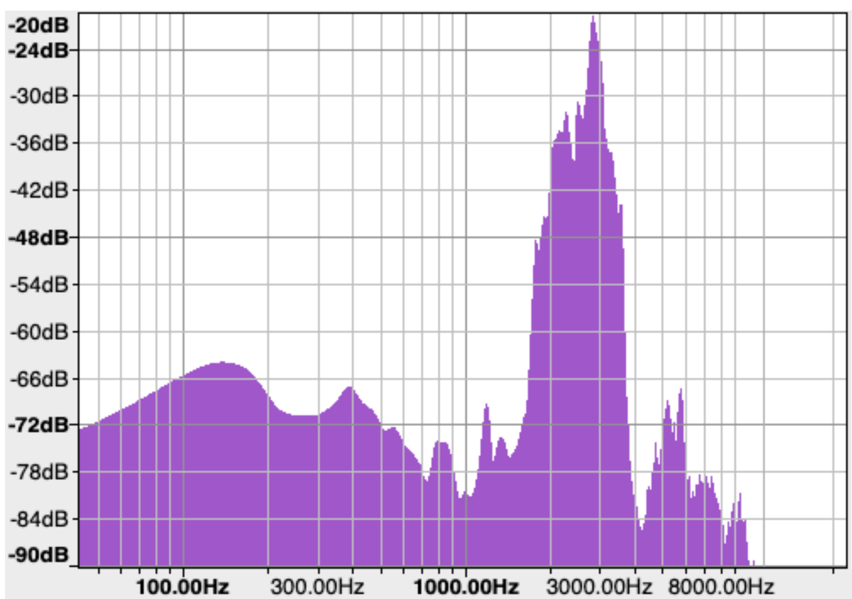
Dolphin 2. Link to Dolphin 2 sounds: http://www.gemarsh.com/wp-content/uploads/Dolph2.wav



Dolphin 2 Signal and Spectrogram. Signal is listed as being ~2.5 sec long. The spectrum of each of the three signals can be seen to be somewhat different.

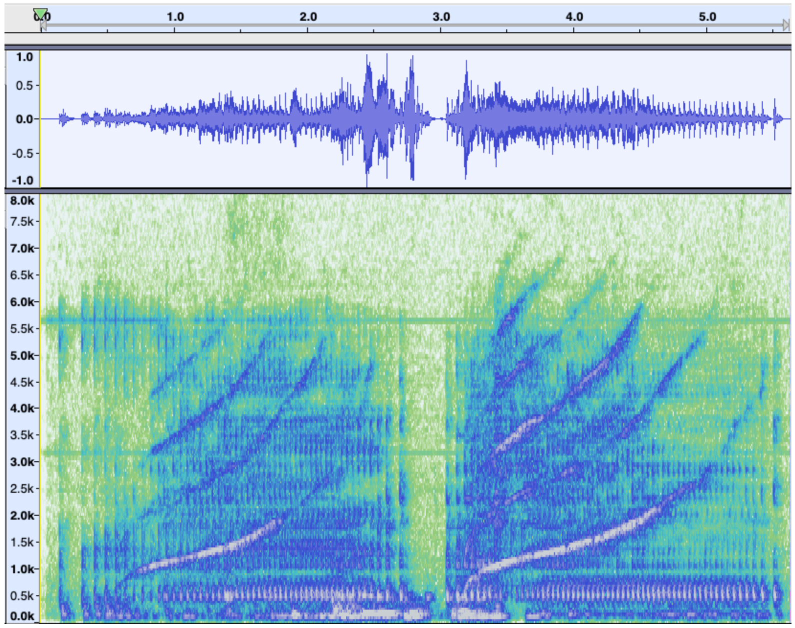


Magnified Portion of the Dolph2 Signal. The frequency corresponding to the interval between the peaks at 0.9242 s and 0.9272 s is 333 Hz.

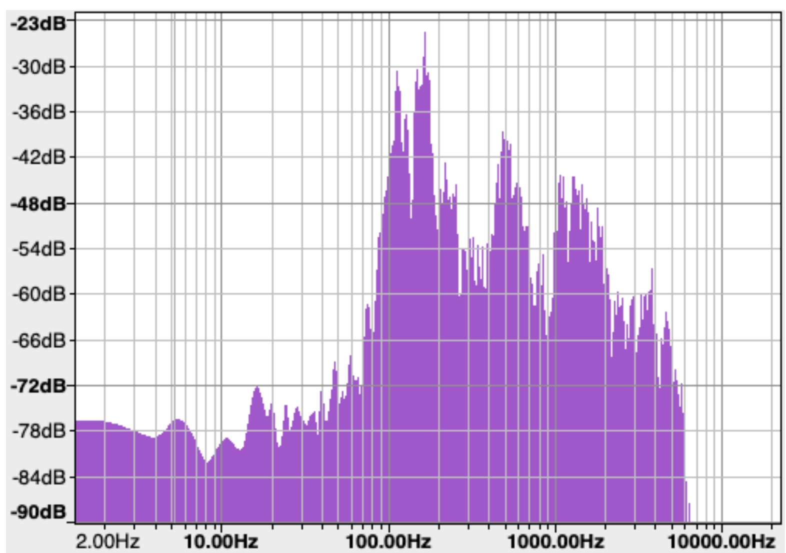


Dolphin 2 Spectrum. The first low frequency peak is at about 140 Hz.

Dolphin 3 An example of a very different type of call. Link to Dolphin 3 sounds: http://www.gemarsh.com/wp-content/uploads/Dolph-3.wav



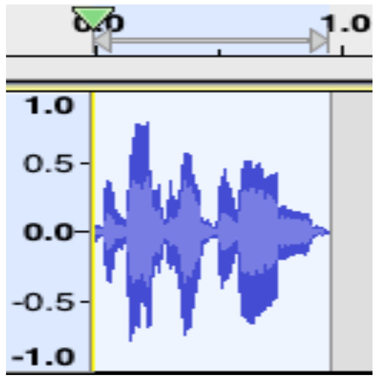
Dolphin 3 Signal and Spectrogram. Not the large number of harmonics.



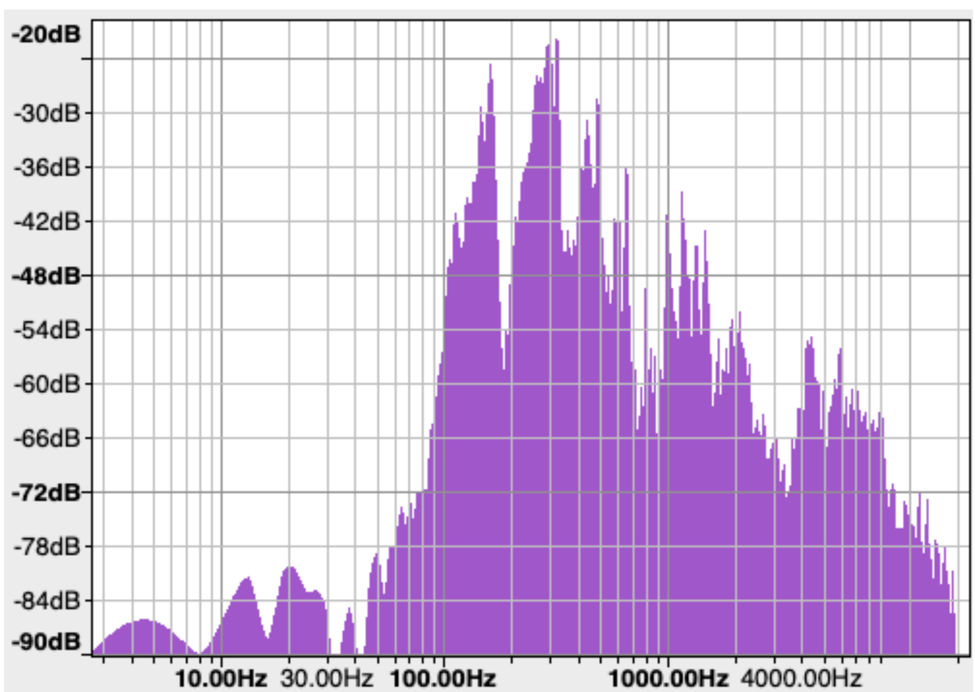
Dolphin 3 Spectrum. Note the peaks between 100 Hz and 200 Hz.

For comparison, the following is for a human saying the words “twenty-two”. The word “two” begins just before 0.5 sec giving a word frequency of about 2 Hz. Note that the ~0.2 sec interval between peaks corresponds to ~5 Hz the effective temporal envelope frequency.

Link to hear the words spoken: http://www.gemarsh.com/wp-content/uploads/Human-speech-saying-22twenty-two22.wav



The corresponding spectrum is shown below.



The peak at 5 Hz and –86 dB could be said to correspond to the average frequency of speech.

Discussion and analysis of the sounds

Dolphin 1

As one can see from the spectrogram, the high frequency component varies from 1.25 kHz to about 2.5 kHz over a ~1 sec interval except for the last third of the signal where the high frequency component was stretched out to a little over 1.5 sec. The magnified signal shows this high frequency component and its amplitude modulation over a short interval. Measuring the time intervals along the peaks of the modulation in the magnified signal gives frequencies in the range of 30-90 Hz. Similar measurements along the peaks visible in the unmagnified signal plot give frequencies in the range of 1.6-5 Hz. The plot of the Dolphin1 spectrum gives a more complete picture. There the loudest peak at -21.1 dB is at 1.43 kHz followed by one at –26.5 dB and 2.4 k Hz. The loudest low frequency peak is at –26.4 dB at 314 Hz followed by those at –30.5 dB at 571 Hz, –35.4 dB at 457 Hz, and –36.7 dB at 214 Hz.

These results are consistent with experiments discussed above that showed that 14 ms tone bursts, amplitude modulated at 600 Hz, evoked a strong auditory neural response that is phase-locked with the envelope of the sound. Here the loudest low frequency peak is at 300 Hz.

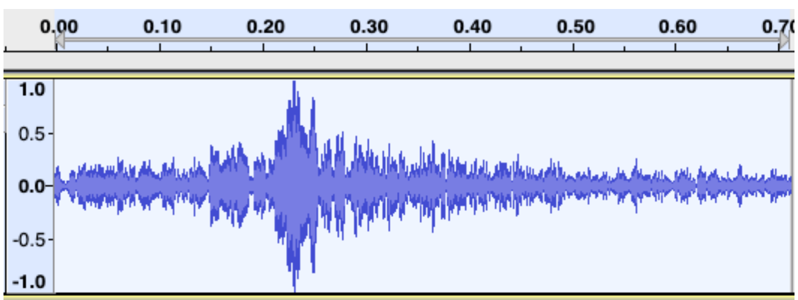
Dolphin 2

As one can see from the spectrogram, the high frequency component varies from 1.8 kHz to about 3.2 kHz over a ~0.5 sec interval. The magnified signal shows this high frequency component and its amplitude modulation over a short interval. Measuring the time intervals along the peaks of the modulation in the magnified signal gives frequencies in the range of 66-357 Hz. Similar measurements along the peaks visible in the unmagnified signal plot give frequencies in the range of 1.3-1.5 Hz. The plot of the Dolphin2 spectrum again shows a more detailed picture. The loudest high frequency peak at –21.1 dB is at 2.82 kHz and is quite narrow. The loudest low frequency peak is at –64.1 dB at 142 Hz followed by one at –67 dB at 383 Hz. It is possible that there is some 60 Hz AC, and multiples thereof with reduced amplitude, that appear in these spectra. 60 Hz is, of course, the frequency of the electric power lines in the U.S.

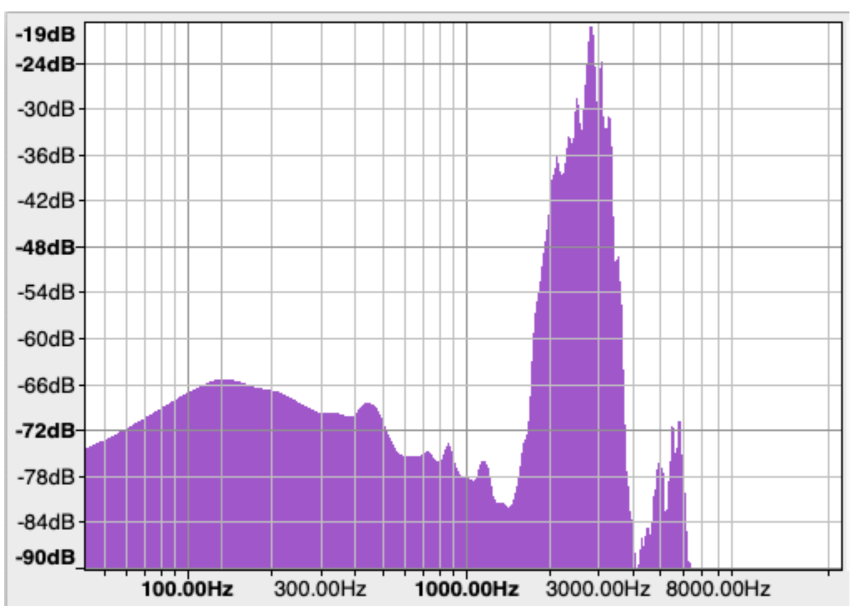
The spectrum between 100 Hz and 600 Hz is also consistent with experiments discussed above. It would appear that the phase locking of the neurological response with the envelope of the sound can occur within this range of frequencies. As is the case in humans, it is likely that for dolphins the primary factor in "speech" intelligibility is the temporal envelope superimposed on the high frequency components of the signal.

Analysis of the 2nd (middle) signal of Dolphin 2

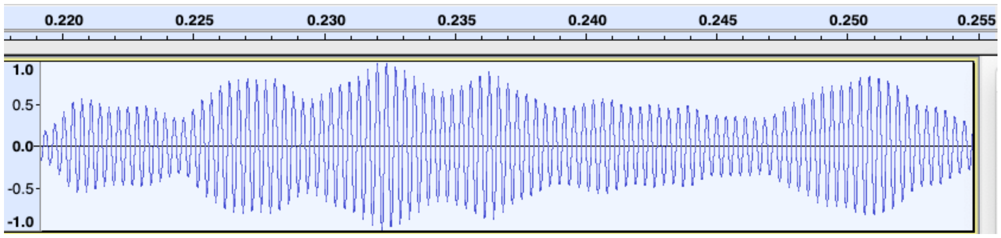
The following is an extended view of the Dolph 2 middle signal, its spectrum and a magnified portion.



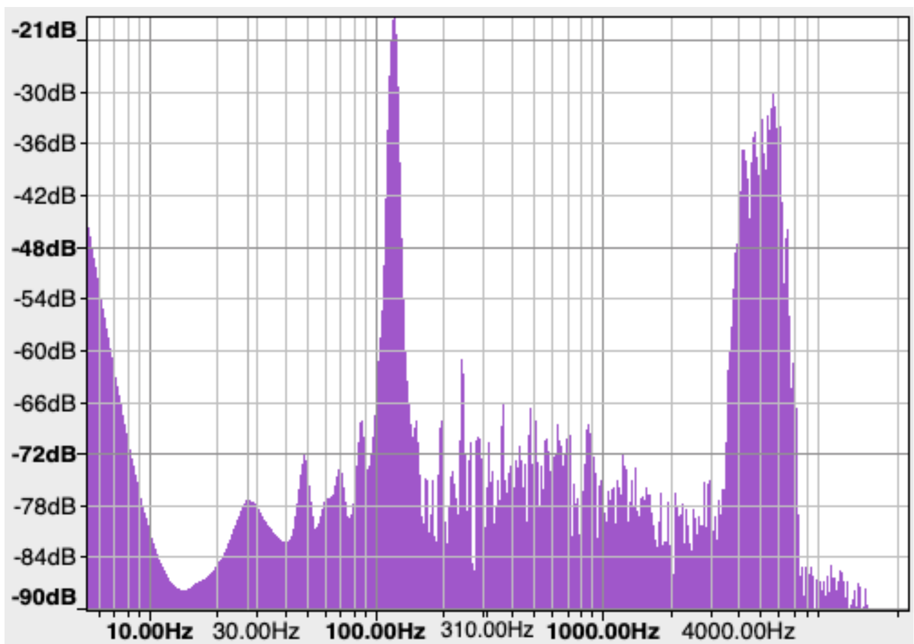
Extended Dolph 2 middle signal.



Spectrum of the extended Dolph 2 middle signal. The peak of the low frequency signals is at 133 Hz followed by the one at 448 Hz.



Magnified portion of the Dolph 2 middle signal. The interval between the low frequency peaks at 0.2275 s to 0.2325 s corresponds to a temporal envelope frequency of 200 Hz.

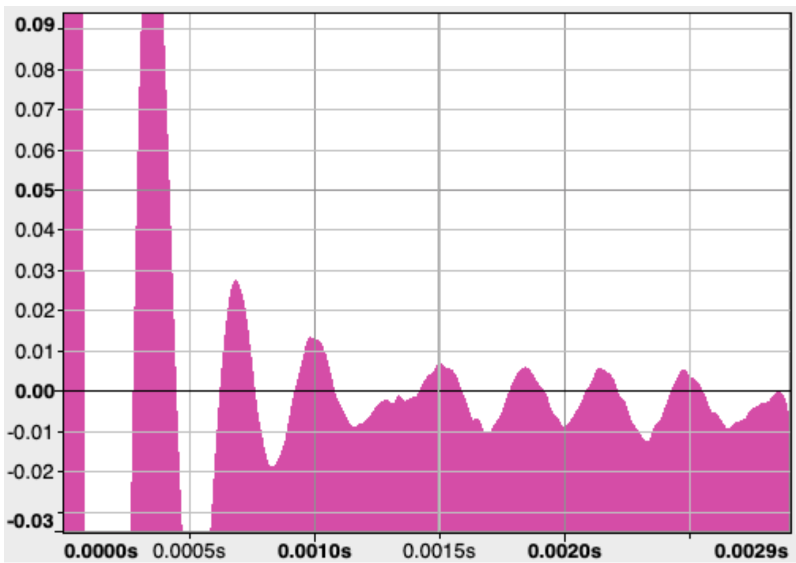


Spectrum of the magnified Dolph 2 middle signal. The center tall peak corresponds to 120 Hz and –21.1 dB. This is probably not a harmonic of 60 Hz AC since there is no peak at 60 Hz.

This again shows that the frequency interval between 100 Hz and 600 Hz is of primary importance for communication. This can also be seen in the cepstrum plot.

The cepstrum is another analysis tool that can be used to obtain an estimate of the spectral envelope. The cepstrum amounts to successively smoothing the Fourier magnitude spectrum to eliminate the high frequency fluctuations. Since the cepstrum can be thought of a lowpass filtering of the curve of the spectrum considered to be a signal, it will average the curve of the spectrum and will not reflect the true envelope, which links the peaks of the curve. One way around this is to combine it with what is called "linear predictive coding" to obtain the discrete cepstrum spectral envelope. It is computed from discrete points in the frequency-amplitude plane from the spectral peaks to generate a smoothly interpolated curve. This will not be considered further here.

In the case of the Dolph 2 middle signal the cepstrum looks like:



The cepstrum of the Dolph 2 middle signal. The abscissa gives the "quefrency", the index of the cepstral coefficients measured in seconds. The plot uses a rectangular window.

Starting from 0.003 s, the major peaks correspond to actual frequencies of 2869 Hz, 1459 Hz, for the high frequencies and 669 Hz, 542 Hz, 468 Hz, 405 Hz, and 351 Hz for the low.

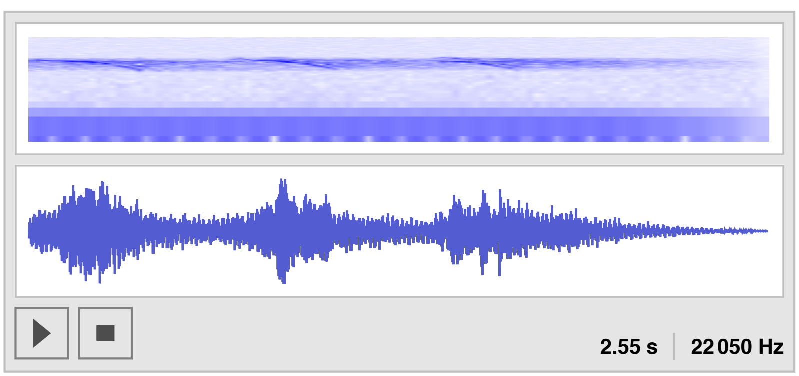
An example of an envelope program that runs in Mathematica is given in the next section.

Use of Mathematica for analysis and reproduction of dolphin sounds

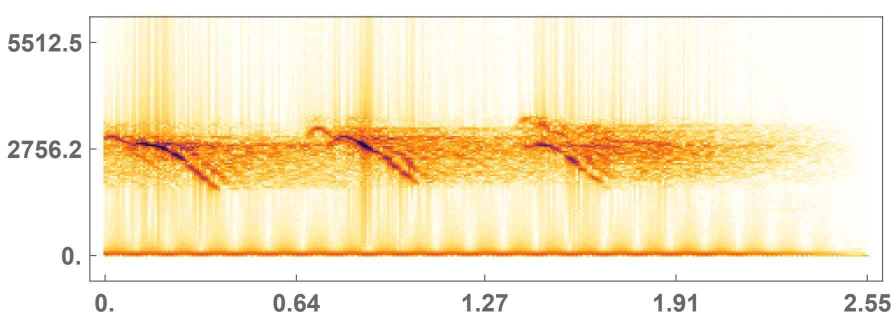
The purpose of this section is to introduce some methods that could be useful in the study of dolphin signals. Another tool that has been used is SIGNAL a sound system produced by Engineering Design on Berkeley, CA.[23] Commonly available WAVE and AIFF format files can be converted for use in SIGNAL. Audacity and Mathematica were chosen to be used here because they are simpler for me to use.

In order to use Mathematica for analysis, one must put the WAV audio files exported from Audacity into Mathematica. This is done by simply dragging them to the Mathematica notebook. While they can be played in the notebook, they cannot be played here. Further on in the paper links are provided to hear the various sounds. In the Mathematica notebook, the WAV files will look like (the waveform is above and the spectrogram below):

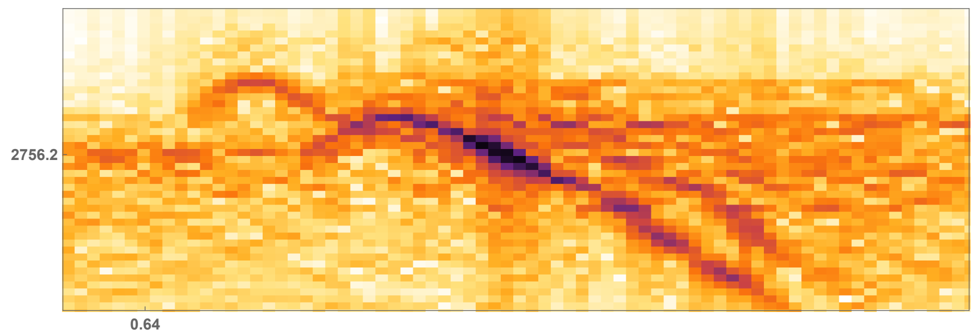
Dolph2 =



Here is a better version of the spectrogram shown above the waveform.



Here is an expanded view of the central portion of the spectrogram:



Mathematica allows one to take points along a curve and produce a numerical file. This was done for the darker curve in the spectrogram above so as to fit a function to the points. The initial value for the time is 0.7779 sec and the last value is 1.052 sec. The resulting fit is the function

$$\text{FIT} = -61578.4 + 215439.\ t - 234403.\ t^2 + 82642.2\ t^3$$

To produce a frequency modulated signal from this one must multiply by 2π. Call this Dolph2FM, i.e., Dolph2FM = 2π FIT, and define the phase as a function of *t* as

$$y = Sin[\int_{0.7779}^{t} (Dolph2FM)\ dt].$$

Now create a new table, call it "*whistle 2 table*", as

whistle 2 table = Table[y , {t, Range[0.7779, 1.052, .00003]}];

The number of entries is determined by the last number in "Range" and in this case the *whistle 2 table* has 9137 entries. A spectrogram of *whistle 2 table* can be created using

Spectrogram[*whistle 2 table*, SampleRate -> 34200, PlotRange -> {{0, 0.27}, {0, 5000}}]

The result looks the same as the original spectrogram so that we have a function that faithfully represents the original.

The data for the central signal of Dolph2, call it Dolph2centerSignal, can be exported to a txt file, call it z.txt, which can then be changed to a simple list of numbers. The way this is done is to use

Export["Dolph2centerSignal.txt", Dolph2centerSignal].

An option "open" appears when this is done and clicking on it gives: SystemOpen["*whistle 2 table*.txt"] and a screen appears with the numbers. This list can be modified as needed by copying the numbers and pasting into MS Word where the find and replace function can be used to add commas, etc. This list is then entered as *plotpnts* = the list of numbers. Don't add brackets or a semicolon. Using ListPlot[*plotpnts*, Joined -> True]results in a plot that looks the same as the original central Dolph2 signal.

To obtain the envelope of this signal we need an approximate smooth function, which can be obtained using interpolation,

f = Interpolation[*plotpnts*, InterpolationOrder -> 7]

This results in

InterpolatingFunction[Domain: {{1., 7.68×10^3}} Output: scalar].

This can be plotted using GraphicsRow[{Plot[f[x], {x, 0, 7671}]}]

It shows that the interpolation is faithful to the original signal.

The envelope can be found using the Mathematica program given below. In that program, g(x) gives the upper envelope and −g(x) the lower. Note that the envelope is symmetric unlike that of the example given above for human speech.

The data points from the plot of the upper envelope, call them envdatapnts , can be obtained by using:

envdatapnts = Cases[Cases[InputForm[envdataup], Line[_], Infinity],

{_?NumericQ, _?NumericQ}, Infinity];

These data points give the amplitude of the upper envelope, but they also have the time coordinate. The amplitude can be separated from the file by the use of:

envdataAmp = Flatten[envdatapnts /. {a_, b_} :> {b}];

(:> here stands fort the special character in Mathematica that appears when \[RuleDelayed] is entered or by typing esc:>esc. It represents a rule that transforms lhs to rhs, evaluating rhs only after the rule is used.)

If *whistle 2 table* defined above is combined with envdataAmp using

prodtable = Flatten[Table[*whistle 2 table* [[t]] envdataAmp[[t]], {t, 1, 7680, 1}]];
prodtableplot = ListPlot[prodtable, Joined -> True, PlotRange -> {{0, 7680}, {-1.5, 1.5}}],

the result is a synthesized version of the original Dolph2 central signal. One can hear what it sounds like by using ListPlay[prodtable, SampleRate -> 44100], which results in an approximation to the original signal. The sound can be heard by clicking on the link: http://www.gemarsh.com/wp-content/uploads/Envhfreq-track6.wav

This is by no means as crisp as the original signal, but is at least a recognizable approximation.

Mathematica envelope program

The following gives an example of an amplitude modulated FM signal and introduces the Mathematica program used to obtain the envelope of a function. The envelope program was put on the web by Andrew Moylan:

https : // mathematica.stackexchange.com/questions/27748/elegant - way - of - obtaining - the - envelope - of - oscillating – function.

The analytic expression for an FM modulated signal was introduce above and here an illustrative set of parameters are used to demonstrate the program. The definition of an FM modulated signal was given above as:

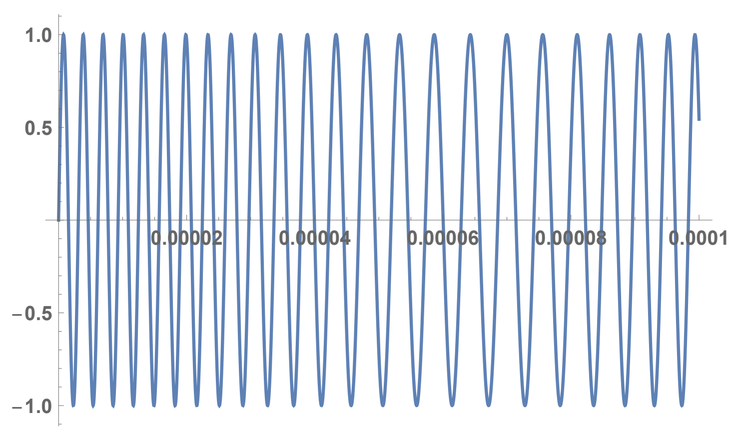
$$\mathrm{FM} = \mathrm{BesselJ}[0, \delta] A_c \mathrm{Sin}[\omega_c t]$$

$$+ \sum_{n=1}^{40} \mathrm{BesselJ}[n, \delta] A_c \{\mathrm{Sin}[(\omega_c + n\omega_m)t] + (-1)^n \mathrm{Sin}[(\omega_c - n\omega_m)t]\};$$

Let:

$\delta = 10;$
$\omega_c = 2\mathrm{Pi}\ 5\ \mathrm{X}\ 10^5\ 0.5;$
$\omega_m = 2\mathrm{Pi}\ 7.5\ \mathrm{X}\ 10^3;$
$A_c = 1;$

$\mathrm{Plot}[\mathrm{FM}, \{t, 0, 10^{-4}\}]$

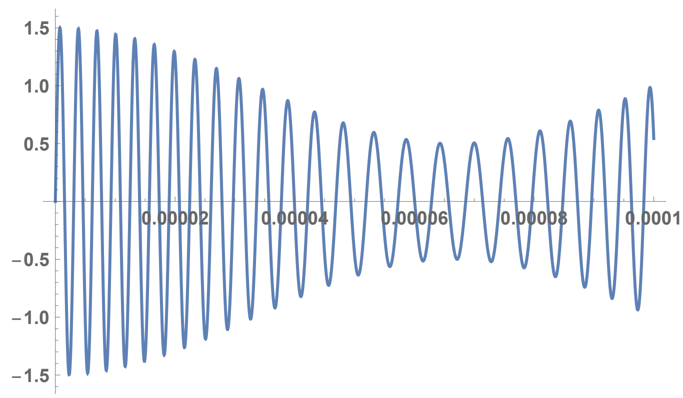


One can see from this figure that the frequency is decreasing with increasing *t*.

The following gives an example of an amplitude modulation of the FM signal:

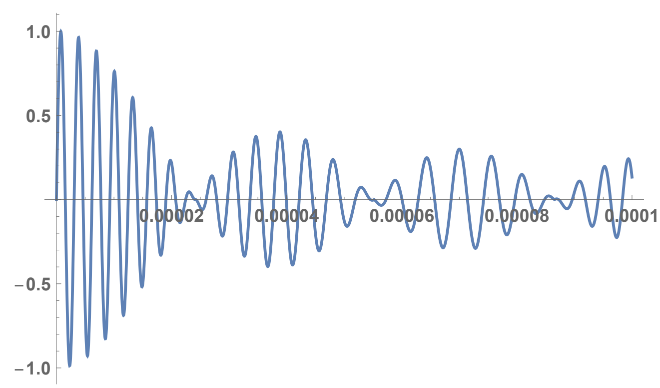
$\omega_c = 2\text{Pi } 5 \text{ X } 10^5 \ 0.5;$
$\omega_m = 2\text{Pi } 7.5 \text{ X } 10^3;$
$k = .5;$

$\text{Plot}[\text{FM}(1 + k\text{Cos}[2\text{Pi } 7.5 \text{ X } 10^3 t]), \{t, 0, 10^{-4}\}]$



Here is another example:

$\text{Plot}[\text{FM Abs}[\text{BesselJ}[0, 10\text{^}5t]], \{t, 0, 10^{-4}\}]$



The envelope of this FM modulated signal can be obtained by using the program:

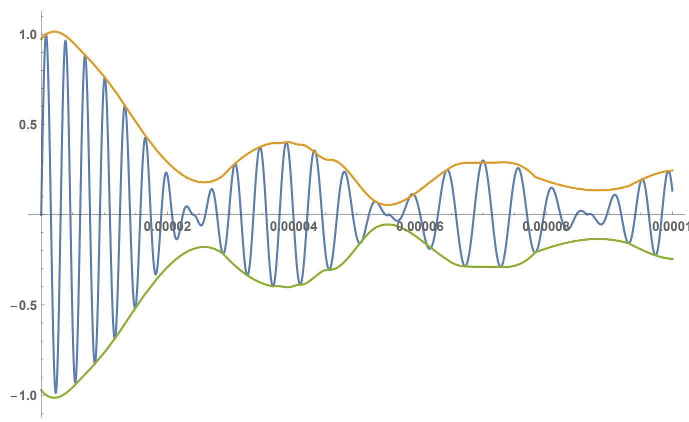
```
f[x_] := FM Abs[ BesselJ[0, 10^5 x]]

FunctionEnvelope[f_, {t_, a_, b_}, n_: 40] :=
 Module[{seeds, x, y, points, progress = 0, tempf, union},
  seeds = Rescale[Range[0, 1, 1/n] + 1/(2 n), {0, 1}, {a, b}];
  points = Last[Last[Reap[Monitor[Do[Quiet@Check[progress++;
           {y, x} = FindMaximum[Abs[f], {t, x0}];
           x = t /. x;
           If[a ≤ x ≤ b, Sow[{x, y}]], Null], {x0, seeds}],
      ProgressIndicator[progress, {0, Length[seeds]}]]]]];
  union[] :=
   points =
    Union[points,
     SameTest ->
           (Abs[First[#1] − First[#2]]/ Replace[Max[Abs[First[#1]], Abs[First[#2]]],
                   u_ /; u == 0 :> 1] < 10^-6 &)];
  union[];
  tempf = Interpolation[points];
  points = Quiet[Join[{{a, tempf[a]}}, points, {{b, tempf[b]}}]];
  union[];
  Interpolation[points]]

g2 = FunctionEnvelope[f[x], {x, 0, 0.0001}, 56];
```

The following plot shows the amplitude modulated FM signal and its envelope:

```
Plot[{f[x], g2[x], −g2[x]}, {x, 0, 0.0001}]
```



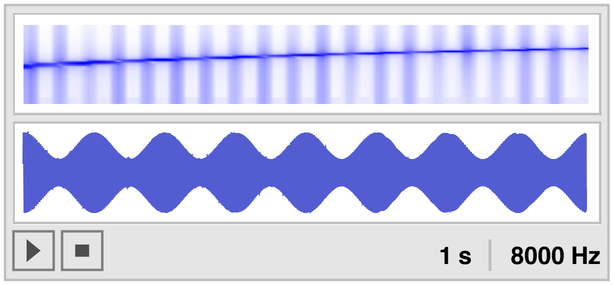
When using this program to obtain the envelope of a dolphin signal, the last number in the expression for g2 = FunctionEnvelope must be adjusted to obtain a smooth envelope. It is also useful to first pass the dolphin signal through a lowpass filter.

Mathematica allows sound to be produced from mathematical expressions. Some examples follow and they can be listened to by clicking on the link for each sound. The Mathematica expression and the Audacity analysis is given for each of the sounds.

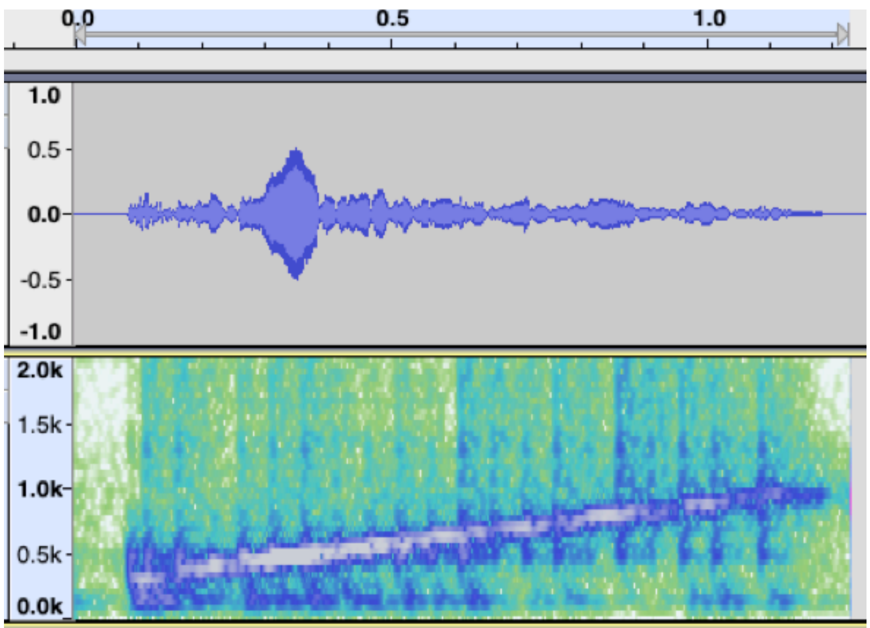
This section shows that a variety of dolphin-like sounds are relatively easy to produce.

sp = Play[(2 + Cos[50 t])*Sin[2000*(1 + Round[2 t, 0.1])*t], {t, 0, 1}]

Round[x] gives the integer closest to x. Round[x,a] rounds to the nearest multiple of a.

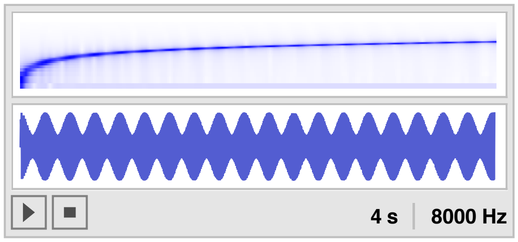


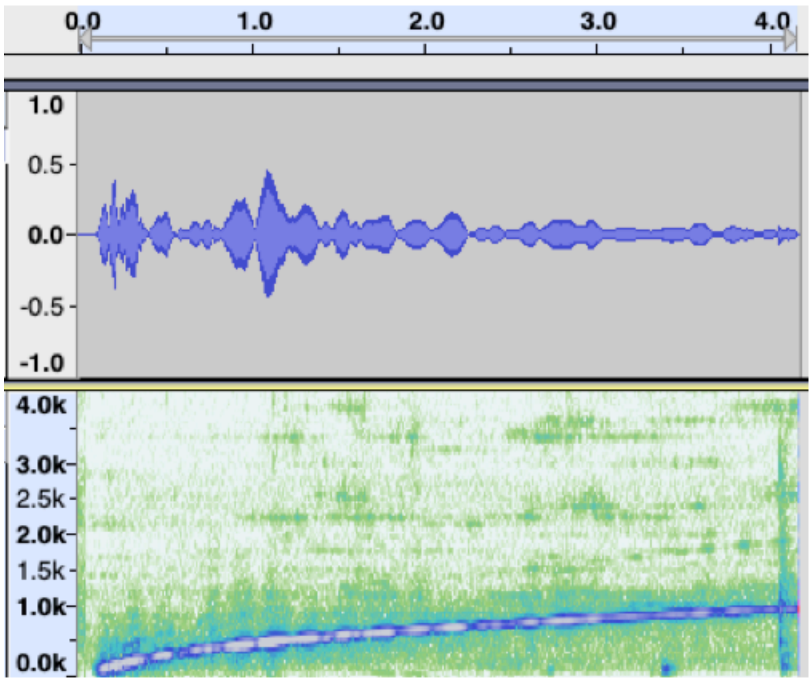
Waveform and spectrogram produced by Mathematica for sp.



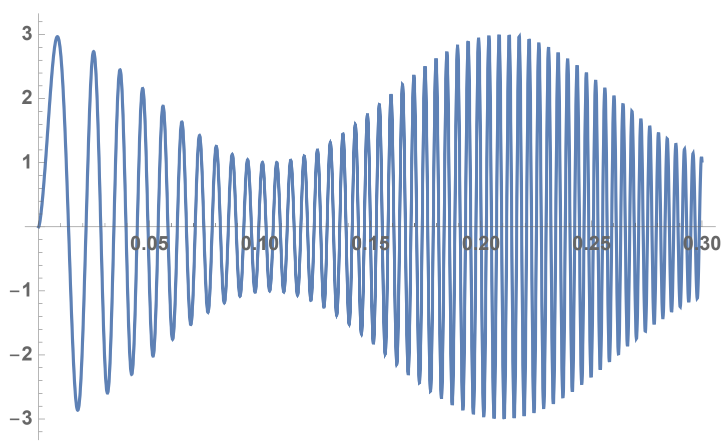
Waveform and spectrogram produced by Audacity for sp.

Link to hear the sound: http://www.gemarsh.com/wp-content/uploads/sp.wav sp1 = Play[(2 + Cos[30 t])*Sin[2000*((1 t)^(3/2))], {t, 0, 4}]

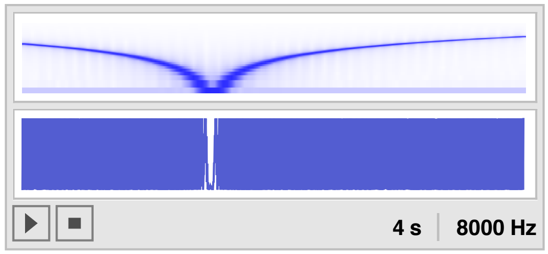


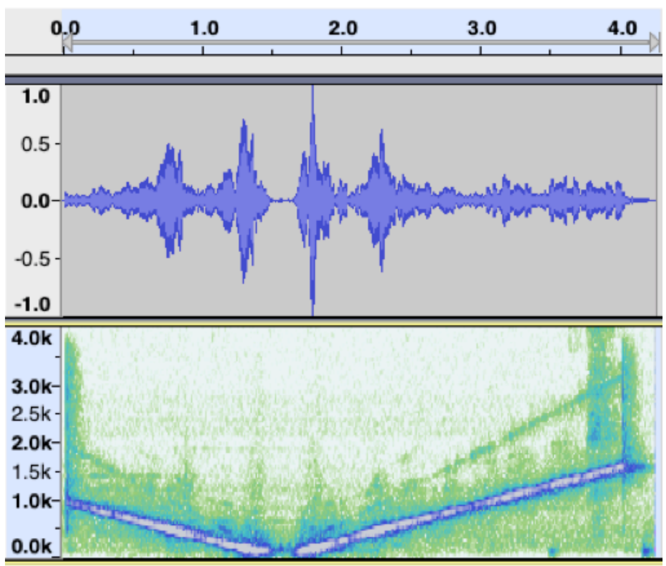


Plot[(2 + Cos[30 t])*Sin[2000*((1 t)^(3/2))], {t, 0, .3}]

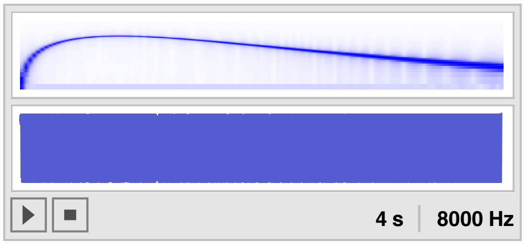


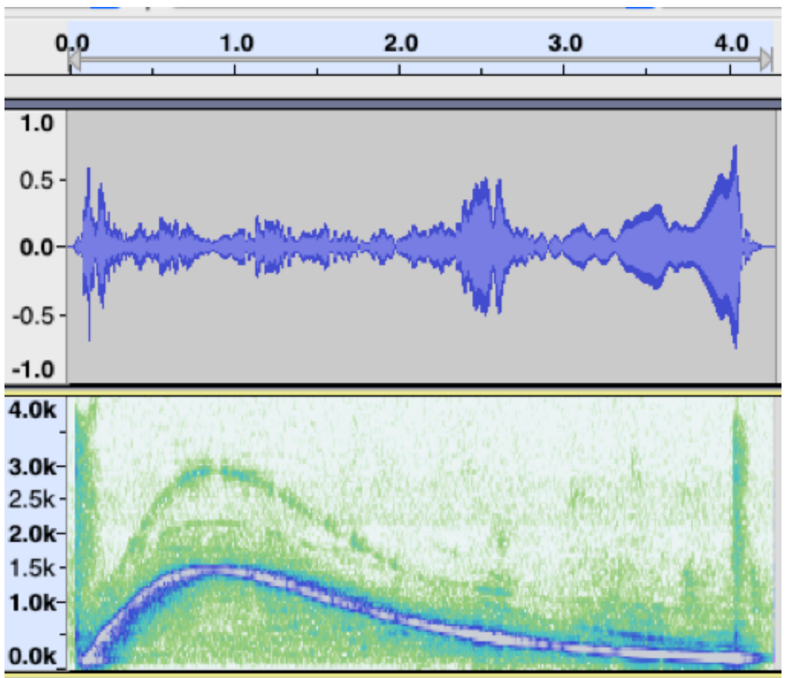
Link to hear the sound: http://www.gemarsh.com/wp-content/uploads/sp1.wav sp2 = Play[Cos[2000 (2 + t^2 - 3 t)], {t, 0, 4}]



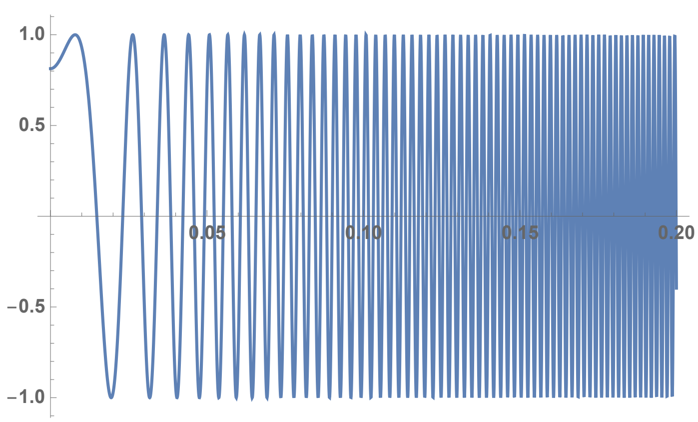


Link to the sound: http://www.gemarsh.com/wp-content/uploads/sp2.wav sp3 = Play[Cos[4000 (10/(t^2 + 2))], {t, 0, 4}]

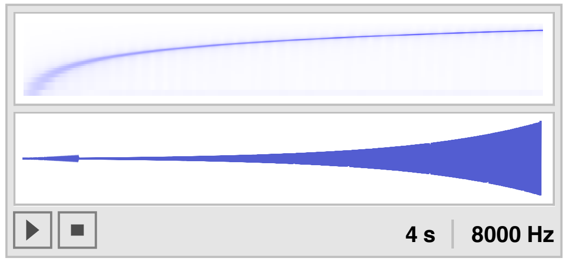


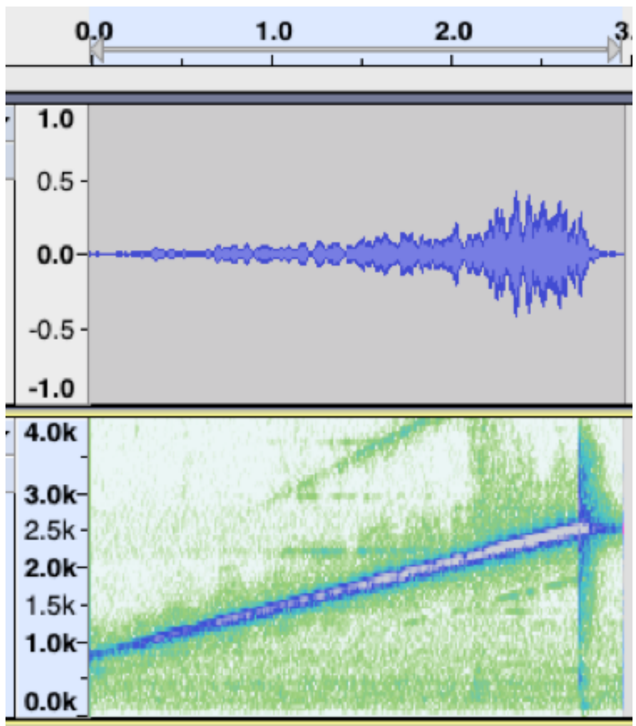


Plot[Cos[4000 (10/(t^2 + 2))], {t, 0, .2}]

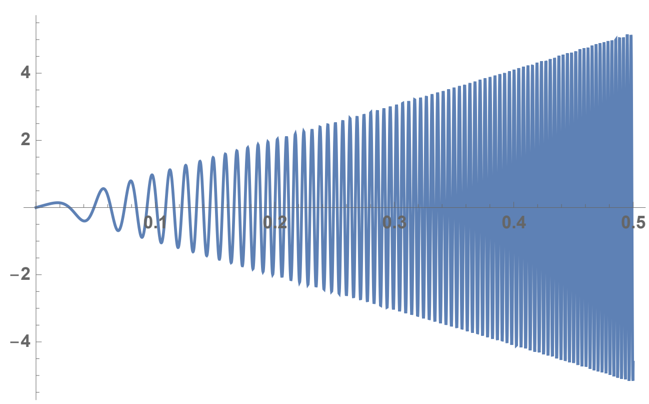


Link to hear the sound: http://www.gemarsh.com/wp-content/uploads/sp3.wav sp4 = Play[Cos[2000 t^2] *5 (Exp[t] - Exp[- t])/1, {t, 0, 4}]





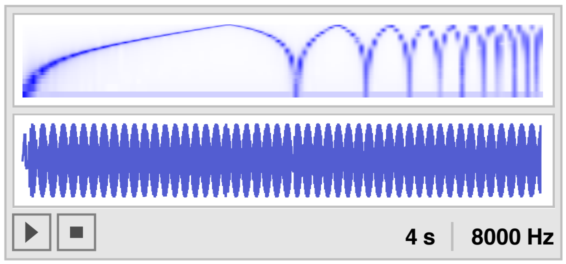
Plot[Cos[2000 t^2] *5 (Exp[t] - Exp[- t])/1, {t, 0, 0.5}]

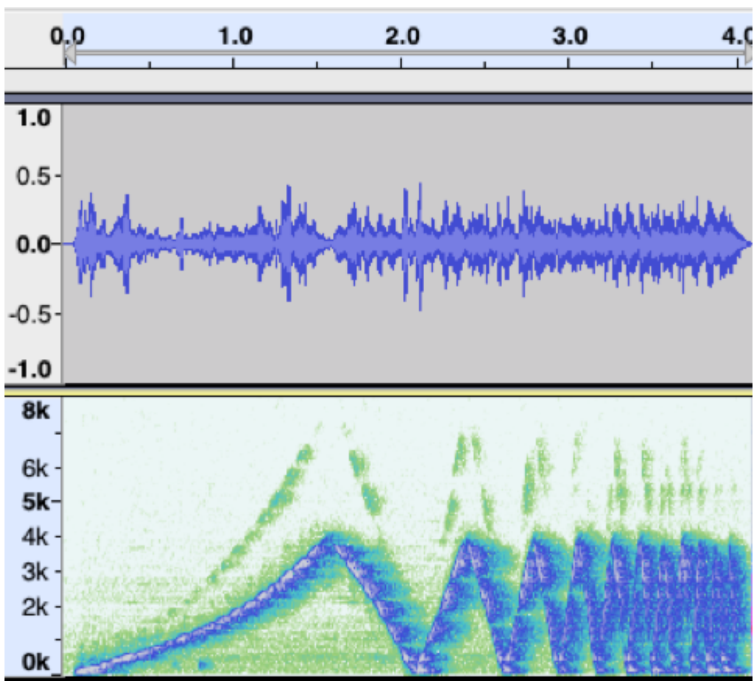


Link to hear the sound:

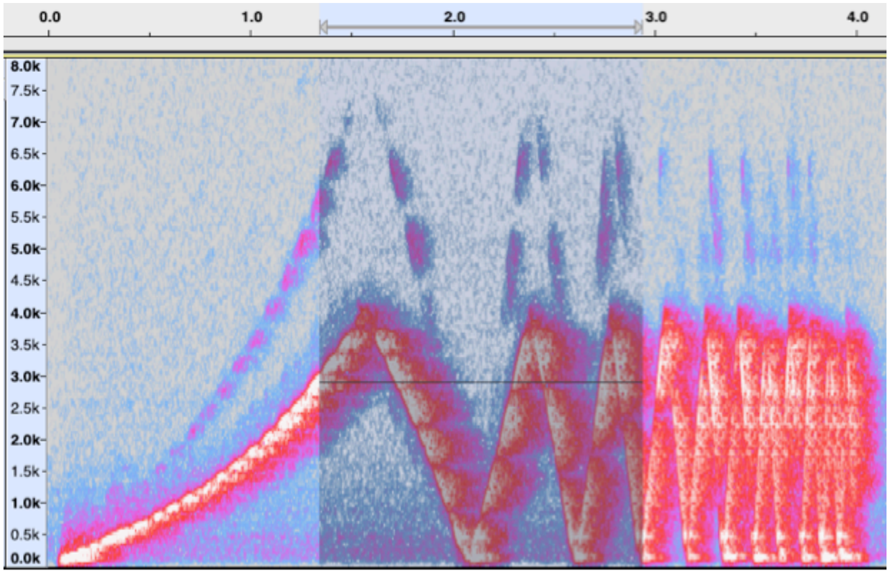
http://www.gemarsh.com/wp-content/uploads/sp4threal-sp4.wav

sp5 = Play[(10 Cos[40 t])*Sin[2000 t *((Exp[t] – Exp[– t])/1)], {t, 0, 4}]

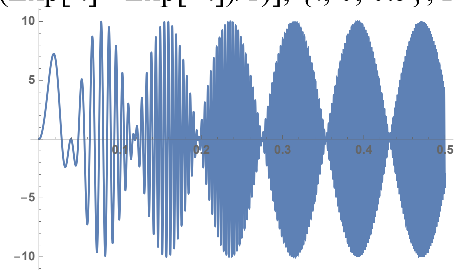




(*The following is a magnified portion of the above*)



Plot[(10 Cos[40 t])*Sin[2000 t *((Exp[t] - Exp[- t])/1)], {t, 0, 0.5}, PlotPoints -> 1000]

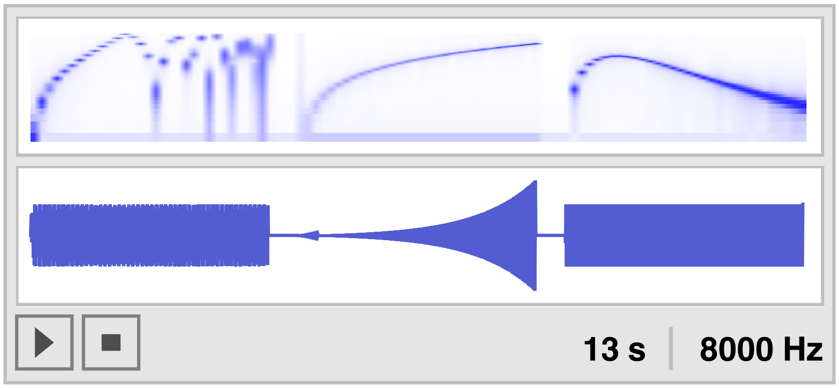


Link to hear the sound: http://www.gemarsh.com/wp-content/uploads/sp5.wav

The sounds can be combined by using:

```
sp6 = Sound[{Play[(10 Cos[40 t])*Sin[2000 t *( (Exp[ t] - Exp[- t])/1)], {t, 0, 4}],
  SoundNote[None, 0.5],
  Play[Cos[2000 t^2] *5 (Exp[ t] - Exp[- t])/1, {t, 0, 4}],
  SoundNote[None, 0.5], Play[Cos[4000 (10/(t^2 + 2))], {t, 0, 4}]}]
```

(*SoundNote[None,0.5] is the delay between sounds in seconds. Magnification of the composite sounds show that all are composed of varying amplitude and frequency sine waves.*)



Link to hear the sound: http://www.gemarsh.com/wp-content/uploads/sp6.wav

The above composite is composed of sp5 followed by sp4 and sp3.

— . . . —

Dolphins and humans

I would like to conclude this paper with a quote from a controversial paper by Vyacheslav Ryabov,[24] who advances the possibility that the interchange of acoustical signals recorded between two quasi-stationary Black Sea bottlenose dolphins constitute a verbal exchange. During the acoustic recording intervals, no special training or food reward was given to the dolphins. The recordings showed that the dolphins took turns in emitting their acoustic signals and did not interrupt each other, showing that each of the dolphins listened to the other's emissions before producing its own response. These responses were not repeats, and the alternate emissions from the dolphins differed in both their spectral components and length. Ryabov maintained that the interchange:

> "resembles a conversation between two people. . . . Since the spoken language of the dolphin consists of spectral extrema that act as phonemes, we can hypothesize that it has both phonological and grammatical structures, so dolphins can create an infinite number of words from a finite number of spectral extrema, which can in turn create an infinite number of sentences. The analysis of the dolphin spoken language in this study has revealed that it either directly or indirectly possesses all the known design features of the human spoken language."

I wrote about Ryabov's paper in a recent book, *The Immense Journey*, saying that "If this study is confirmed, it will place a special ethical and moral imperative on humanity to protect these creatures as intelligent beings. Our excuse for past treatment of all cetaceans—which may have a language similar to the dolphins, that it was unclear that they have high intelligence will no longer hold. In all fairness to humanity, it is very difficult to estimate the intelligence of a creature that is perfectly adapted to its environment. It is important that our understanding of the dolphin language is expanded so that the possibility of direct communication could be explored."

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