# THE ENIGMA OF SATURN'S NORTH-POLAR HEXAGON

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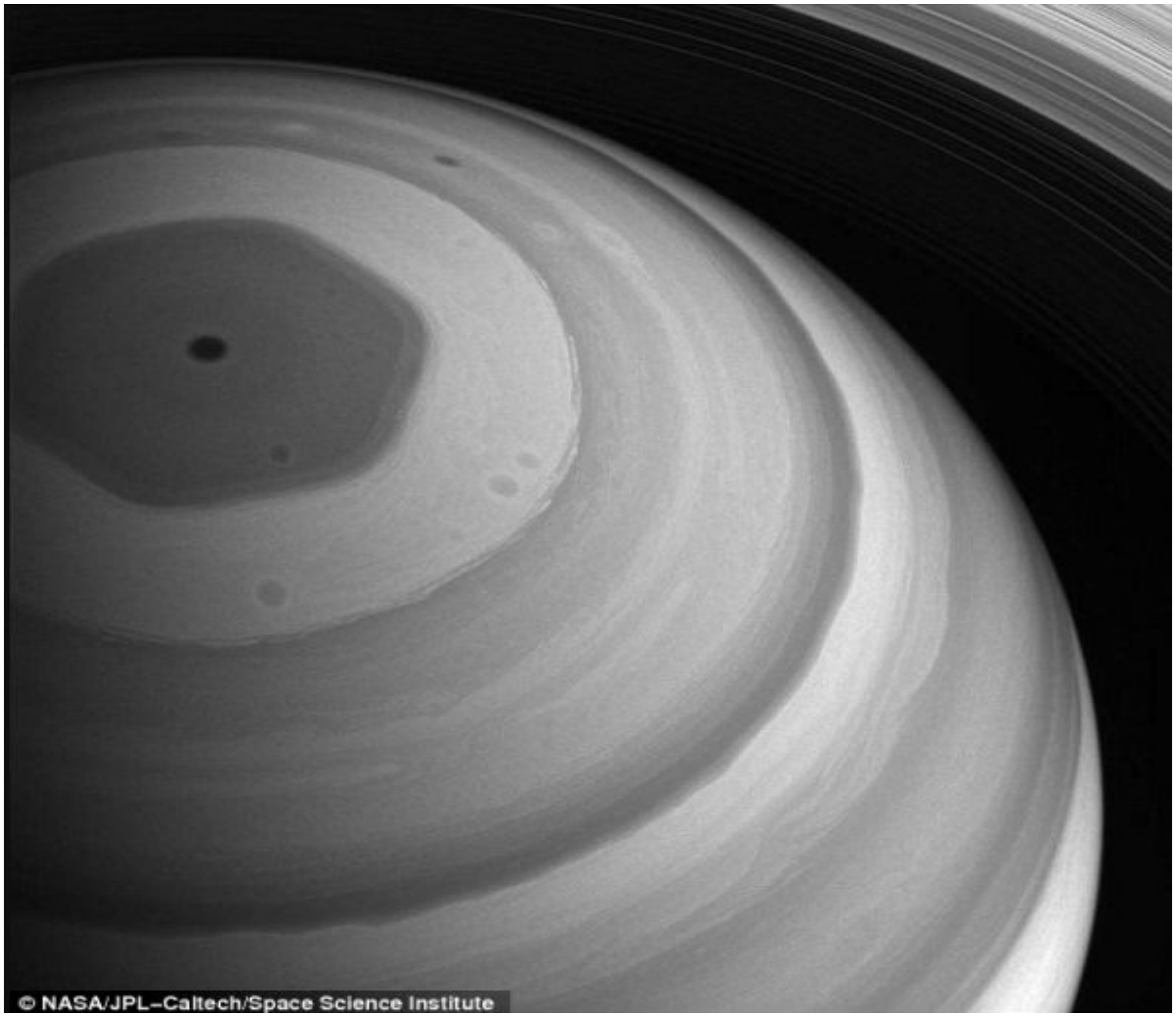
# THE ENIGMA



# NASA HEXAGON VIDEOS

# SATURN

- **GAS GIANT WITH NO SOLID SURFACE**

- **ATMOSPHERE ~75-96% MOLECULAR HYDROGEN AND ~24-3% HELIUM WITH TRACE AMOUNTS OF MOSTLY METHANE AND AMMONIA**

- **AEROSOLS OF AMMONIA ICE, WATER ICE, AND AMMONIA HYDROSULFIDE. IMPORTANT FOR SCATTERING OF LIGHT TO GIVE COLOR CONTRAST**

- **TRANSITION TO METALLIC HYDROGEN OCCURS AT ~40% OF PLANETARY RADIUS**

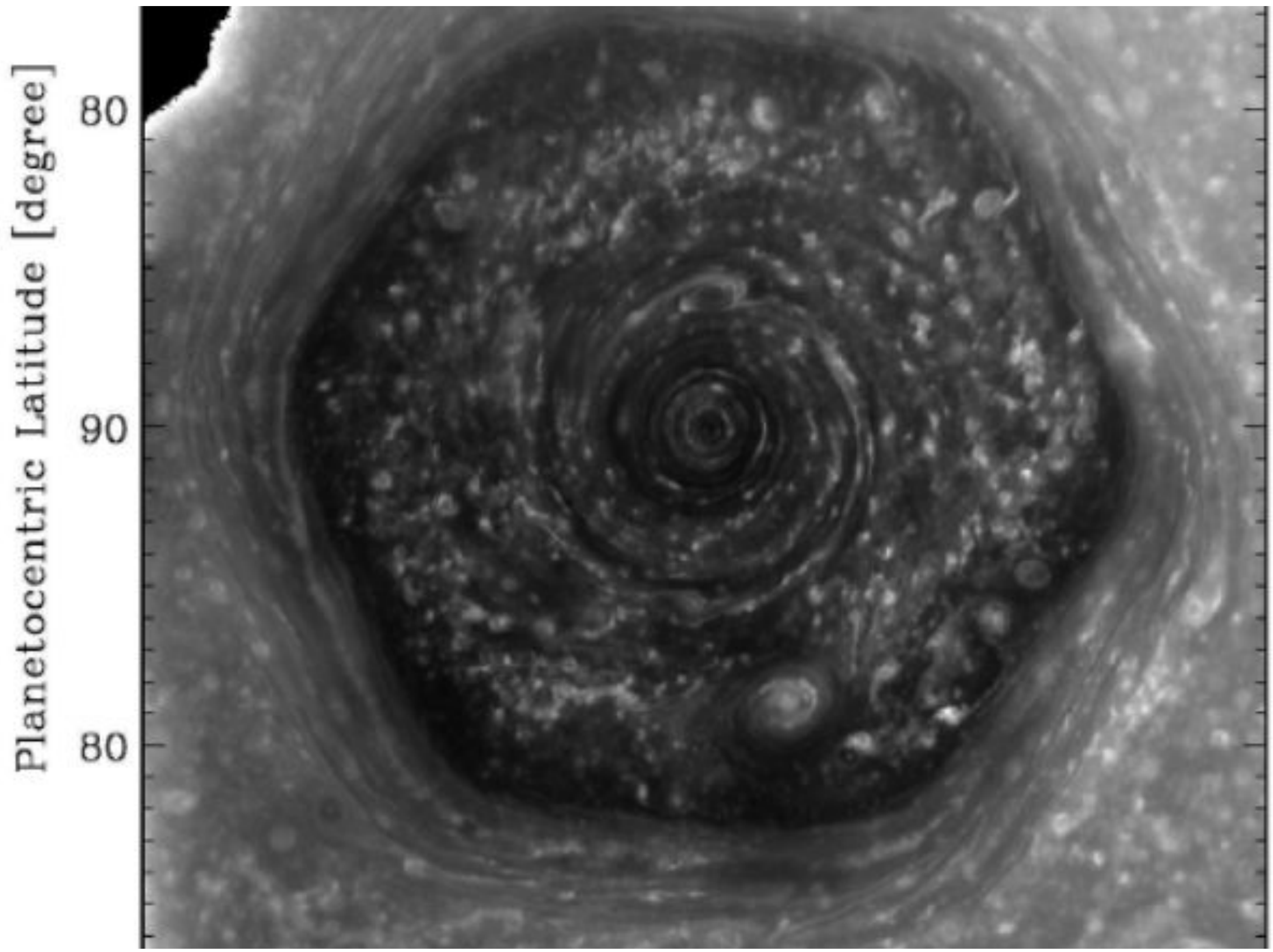
# HEXAGON TEMPERATURE GRADIENT

- UNUSUAL IN THAT THE MERIDIONAL TEMPERATURE GRADIENT HAS THE EQUATORIAL SIDE COLDER THAN THE POLAR. THE HEXAGON IS INSENSITIVE TO RADIATIVE CHANGES OVER A SATURN YEAR.

- THE HEXAGON IS LOCATED AT 76°N AND ROTATES IN A DIRECTION OPPOSITE TO SATURN'S ROTATION. THERE IS ALSO A LESS OBVIOUS HEXAGONAL STRUCTURE AT 79°N THAT ROTATES IN THE SAME DIRECTION AS THE PLANETS ROTATION. THIS HEXAGON IS WARMER THAN THE MORE OBVIOUS HEXAGON AT 76°N. THERE IS A GAP BETWEEN THEM (NEXT SLIDE).

- THIS UNUSUAL GRADIENT MUST HAVE A DYNAMICAL EXPLANATION AND IT HAS BEEN SUGGESTED BY FLETCHER, ET AL. THAT IT MIGHT BE DUE TO AN UPWELLING ON THE EQUATORIAL SIDE OF THE HEXAGON AND SUBSIDENCE ON THE POLAR SIDE. THE DETAILS OF THE PROPOSED CIRCULATION ARE UNKNOWN.

# HEXAGONAL STRUCTURE AT 79$^{o}$N



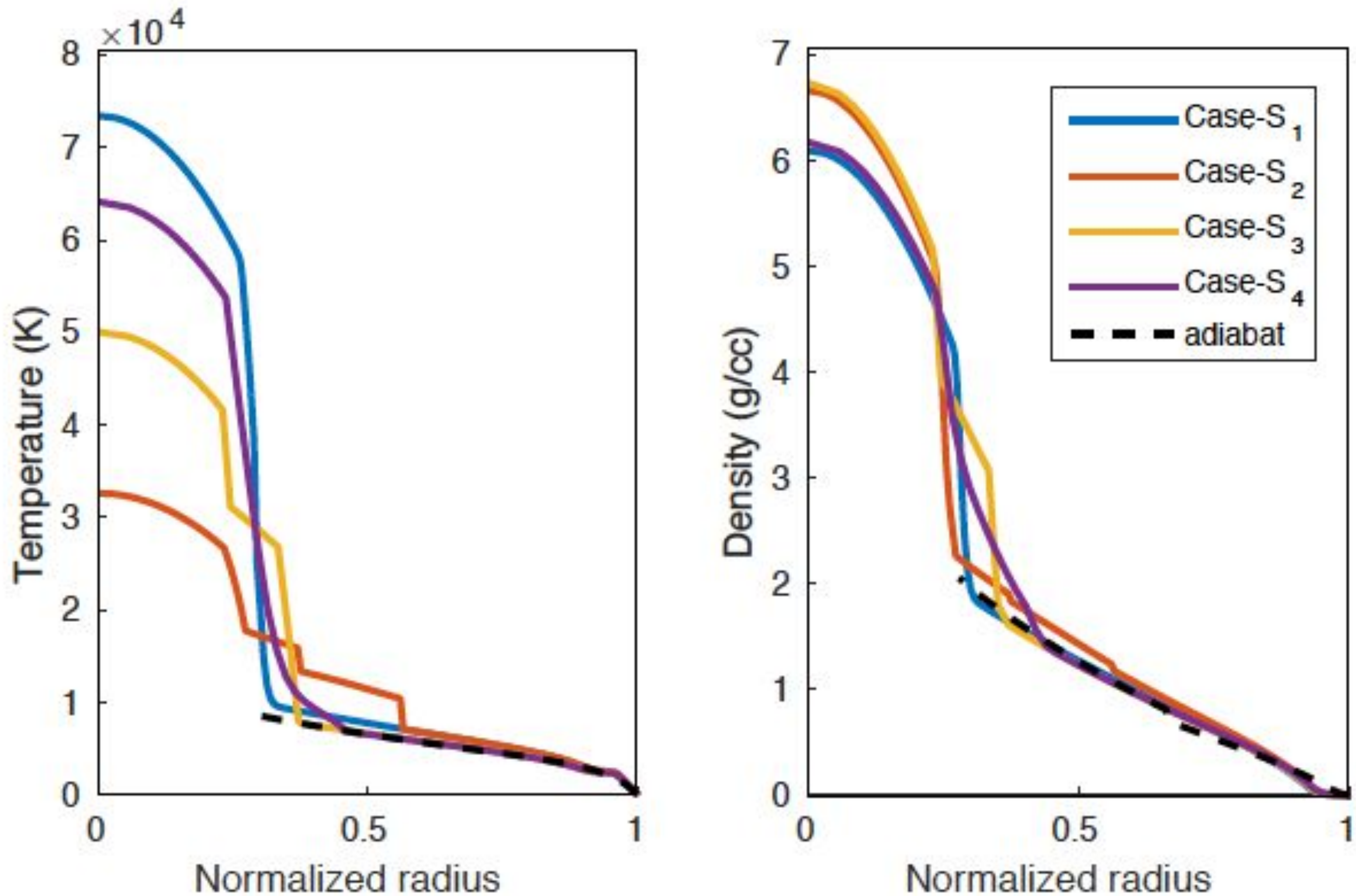

**NOTE THE GAP BETWEEN THE HEXAGON AND THE INTERIOR HEXAGONAL STRUCTURE**

# HEXAGON TEMPERATURE GRADIENT (2)

- THE TEMPERATURE GRADIENT IS STRONGLY DEPENDENT ON THE PRESSURE (DEPTH INTO THE ATMOSPHERE)—THE STRATOSPHERE BEING LARGER THAN THE TROPOSPHERE.

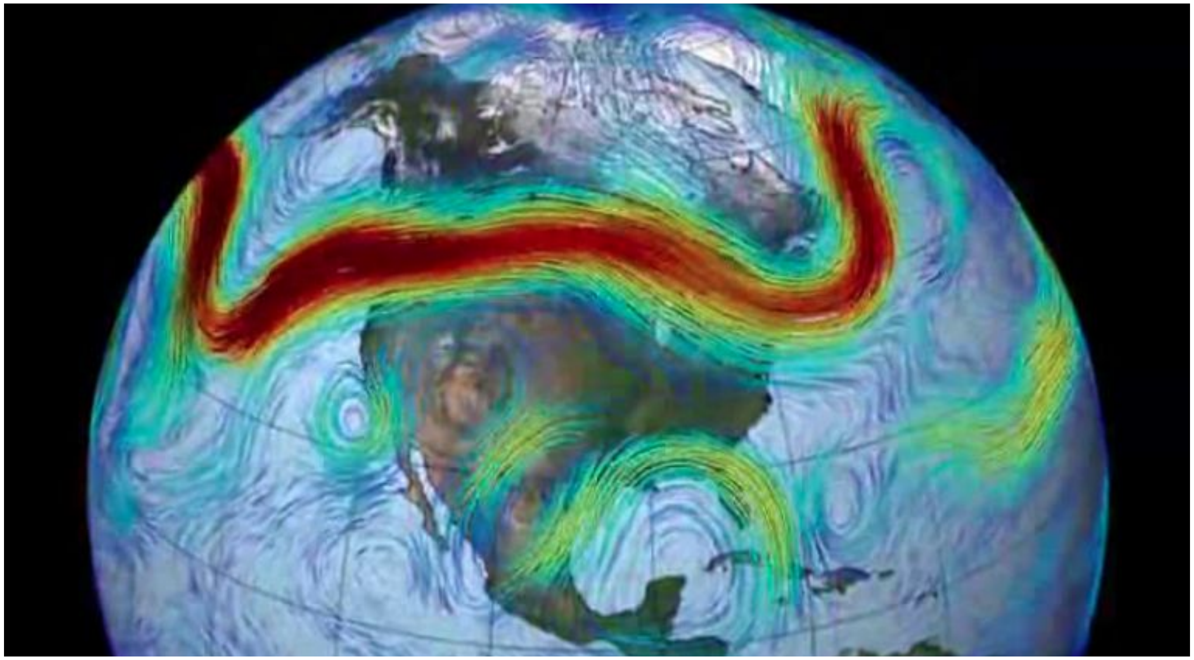
- THE TROPOSPHERIC TEMPERATURE GRADIENT ACROSS THE HEXAGON AT AN ATMOSPHERIC DEPTH OF ~1-2 BAR IS PROBABLY AT MOST 1-2 K

# SATURN TEMPERATURE AND DENSITY PROFILES



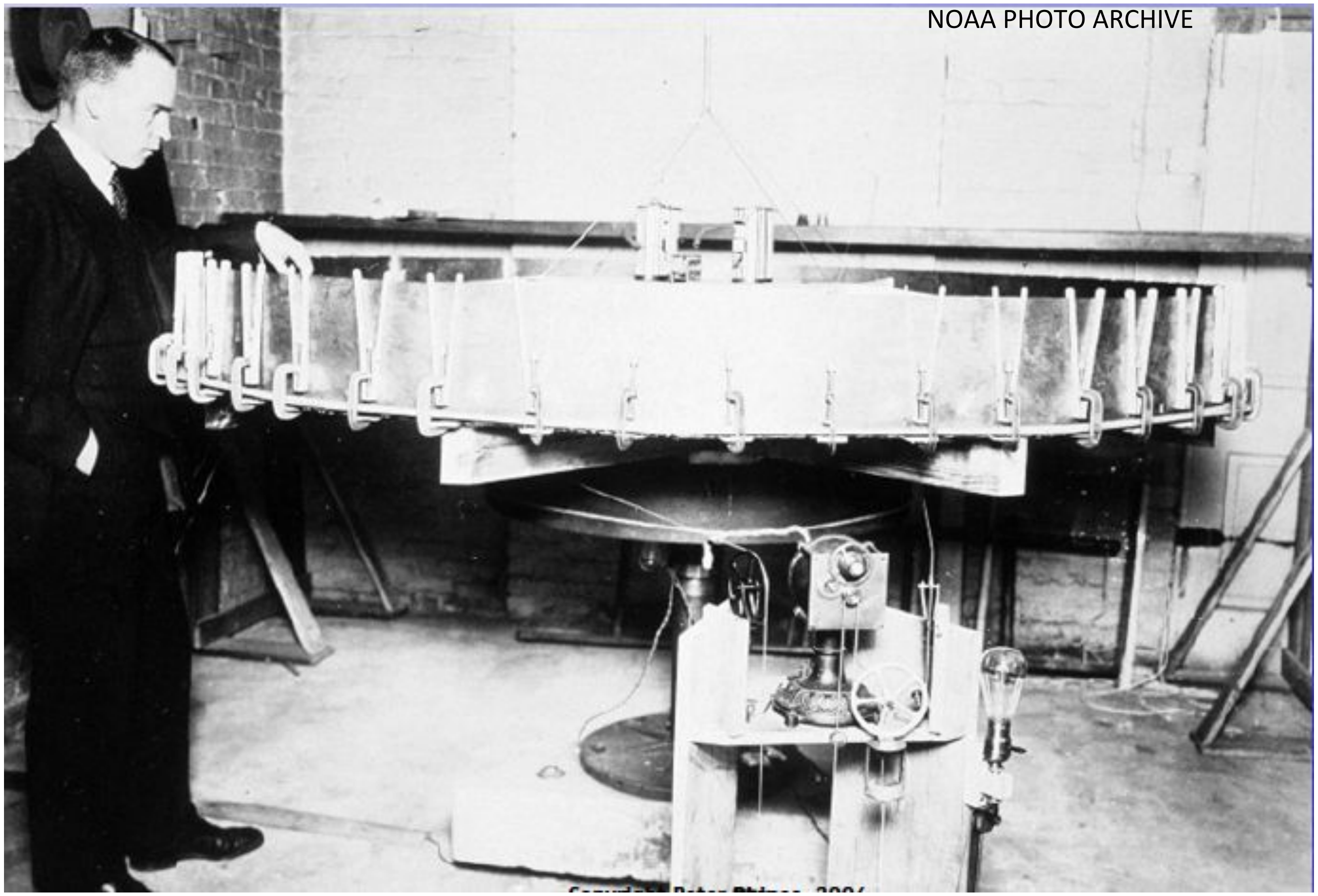

**TEMPERATURE AND DENSITY PROFILES FROM FOUR SATURN MODELS AS WELL AS AN ADIABATIC ENVELOPE MODEL [Vazan, et al., arXiv:1606.01558 (2016)]**

# HEXAGON AS A ROSSBY WAVE



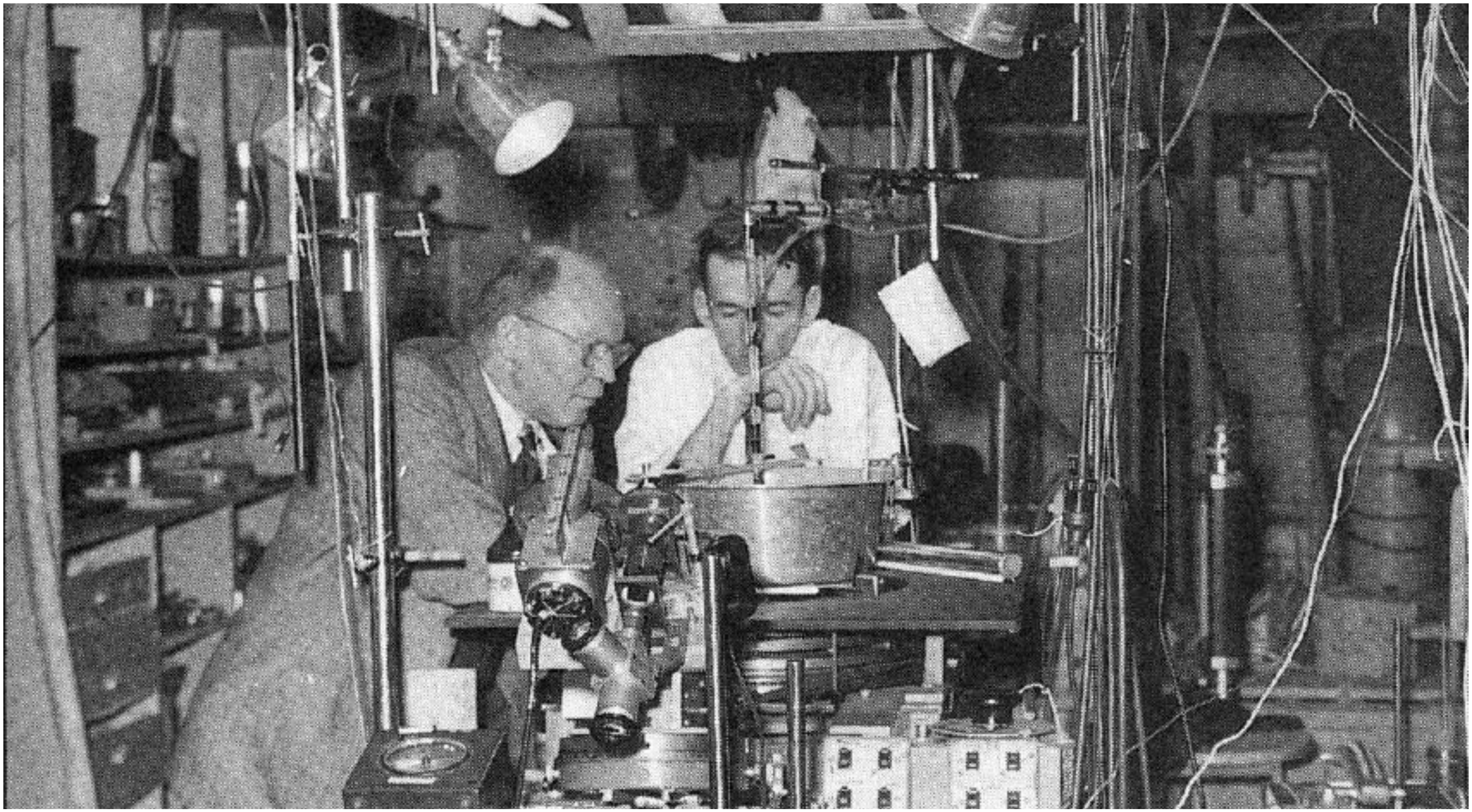
- ALLISON, GODFREY, AND BEEBE SUGGESTED IN 1987 THAT SATURN'S NORTH-POLAR HEXAGON COULD BE A STATIONARY ROSSBY WAVE. A ROSSBY WAVE IS THE MEANDERING OF A JET STREAM.
- HAVE ROSSBY WAVES EVER BEEN OBSERVED IN THE EARTH'S ATMOSPHERE? NO!
- SATURN'S HEXAGON IS PERHAPS THE ONLY CASE OF A DIRECTLY OBSERVABLE ROSSBY WAVE OUTSIDE OF A LABORATORY.

# Carl-Gustav Rossby 1920s



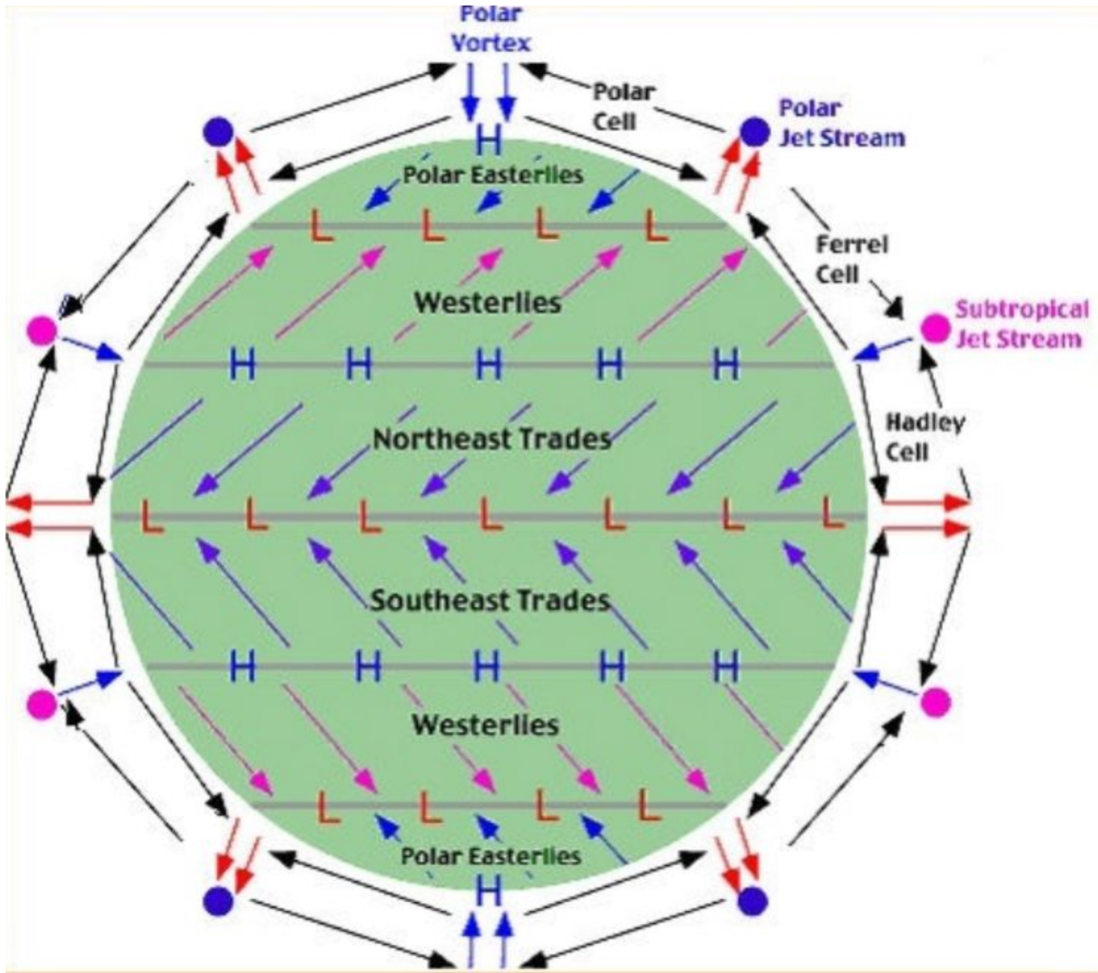

**ROTATING TABLE IN WASHINGTON WEATHER BUREAU BASEMENT COULD HAVE SHOWN HIM ROSSBY WAVES BUT IT COLLAPSED AND WAS APPERENTLY NEVER REPAIRED. ROSSBY DID SEE**
**THEM AT THE FULTZ LABORATORY AT THE UNIVERSITY OF CHICAGO AROUND 1953.**

# CARL ROSSBY AND DAVE FULTZ



**FULTZ LABORATORY IN THE BASEMENT OF ROSENWALD HALL AT UNIVERSITY OF CHICAGO c.1953**

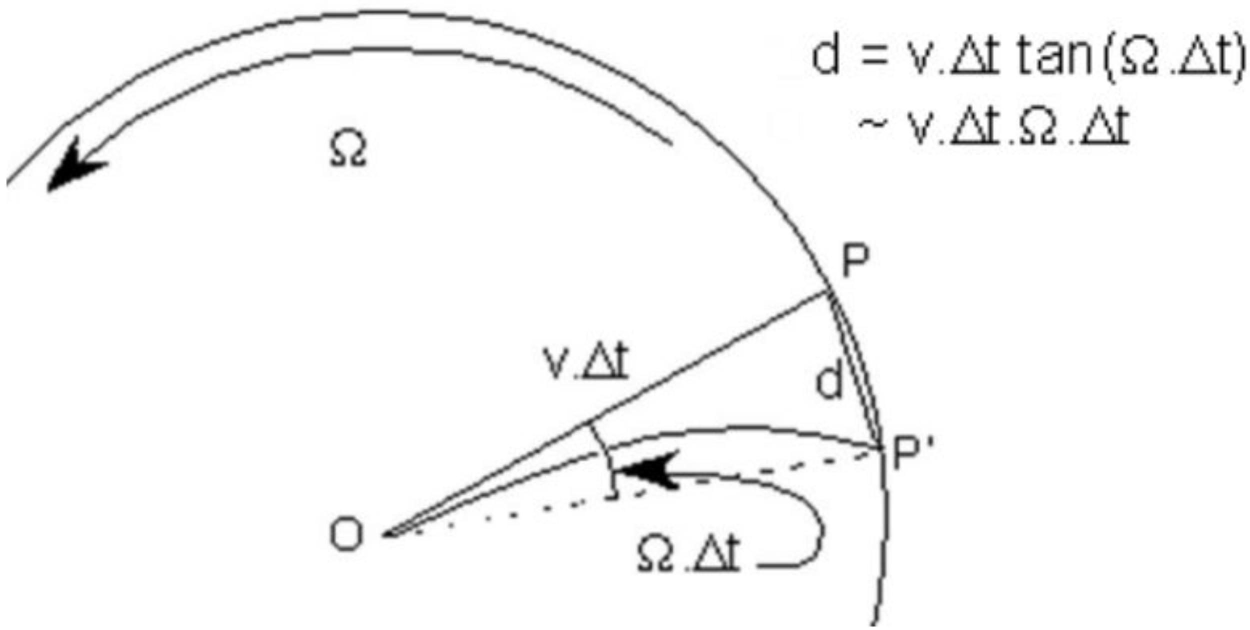
# EARTH JET STREAMS




**“WESTERLIES” FLOW TO THE EAST AND “EASTERLIES” FLOW TO THE WEST!**

**THE PATTERN IS DUE TO THE ROTATION OF THE EARTH AND THE CORIOLIS FORCE**

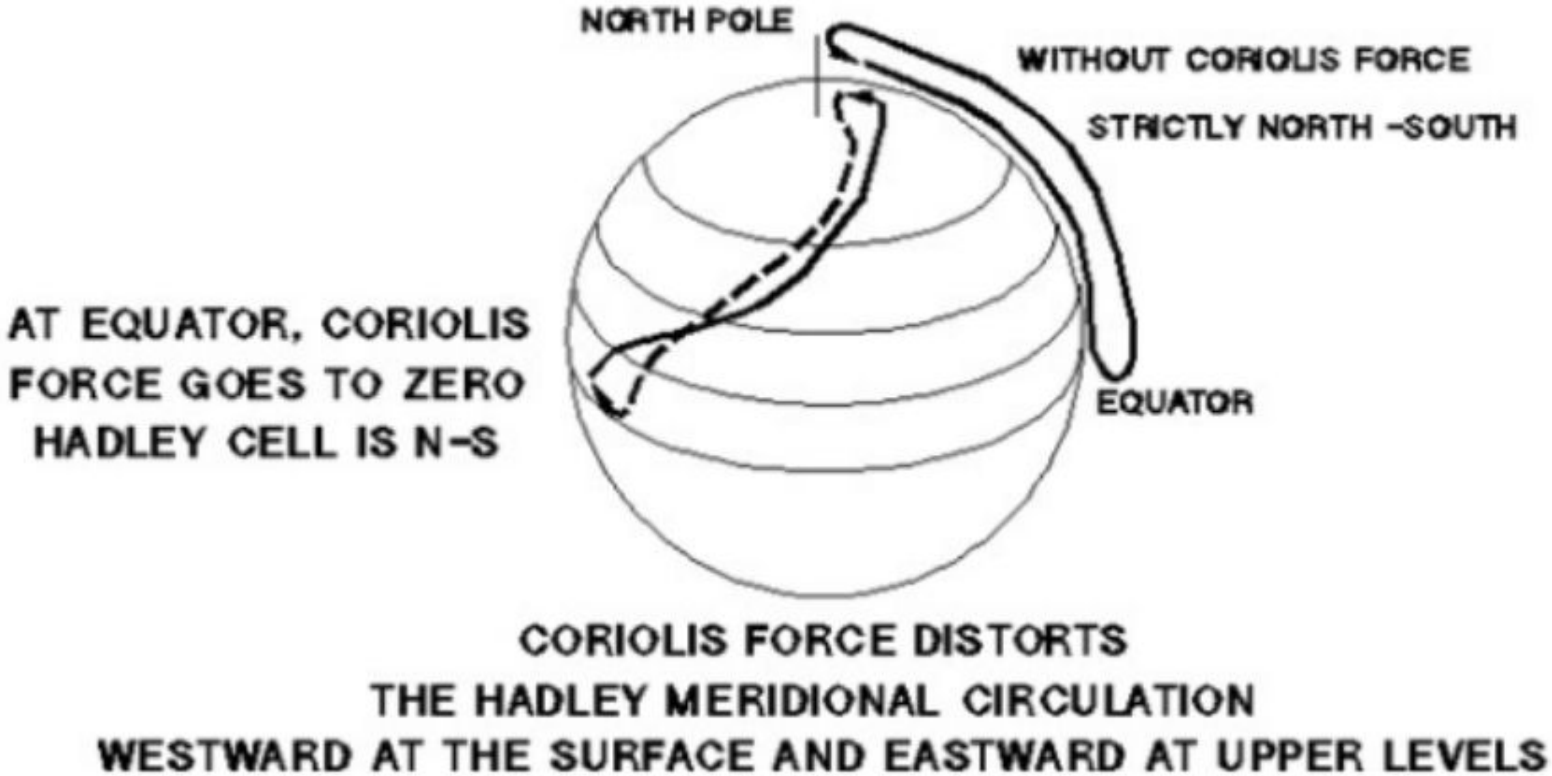
# CORIOLIS FORCE




**IN A ROTATING FRAME OF REFERENCE NEWTON'S LAWS WILL HOLD IF THE FICTITIOUS CORIOLIS AND CENTRIFUGAL FORCES ARE ADDED TO THE TRUE FORCE.**

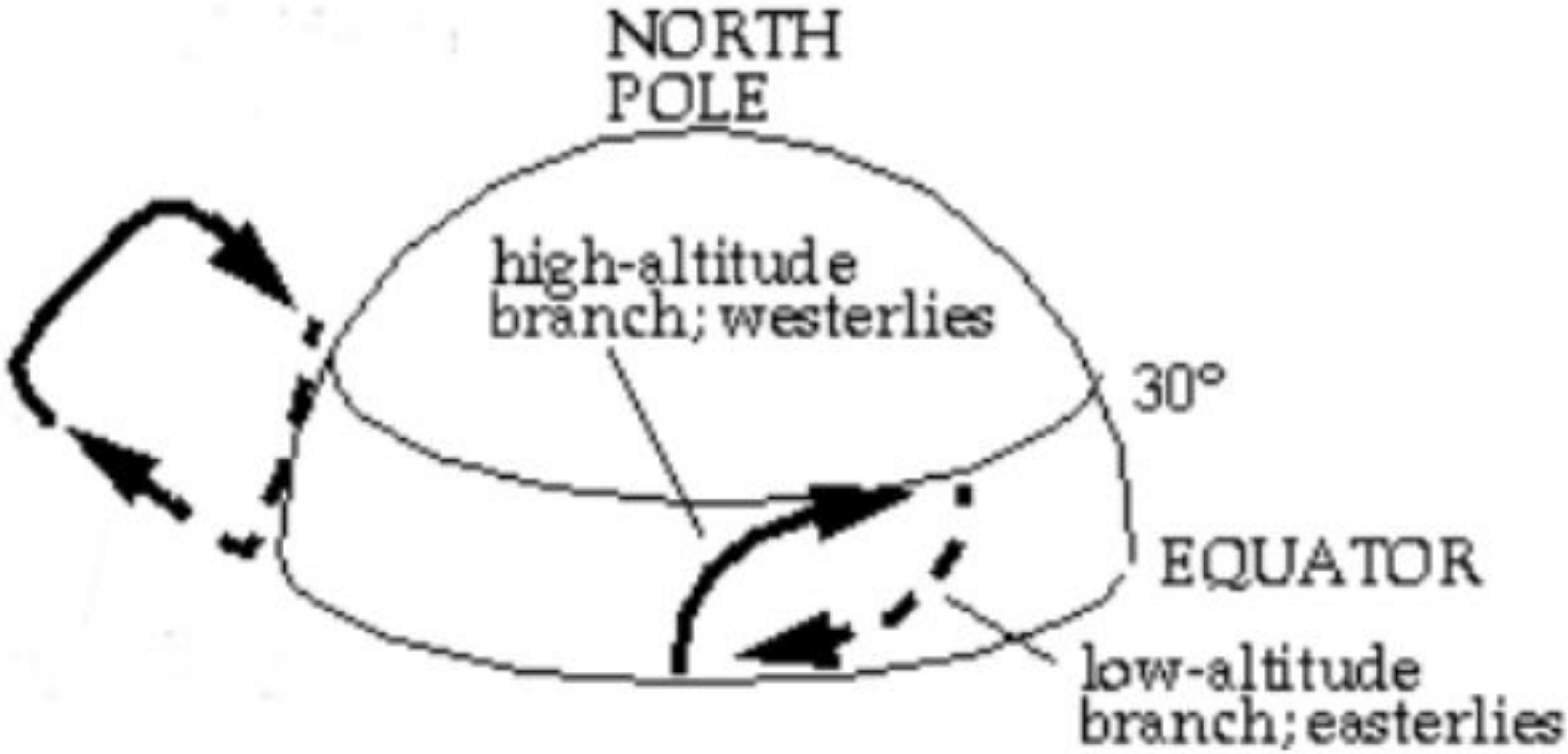
**ON A ROTATING SPHERE THE CORIOLIS FORCE IS $2\Omega V \sin\Theta$. ($\Theta$ = LATITUDE)**

# ACTUAL CELL AIR FLOW

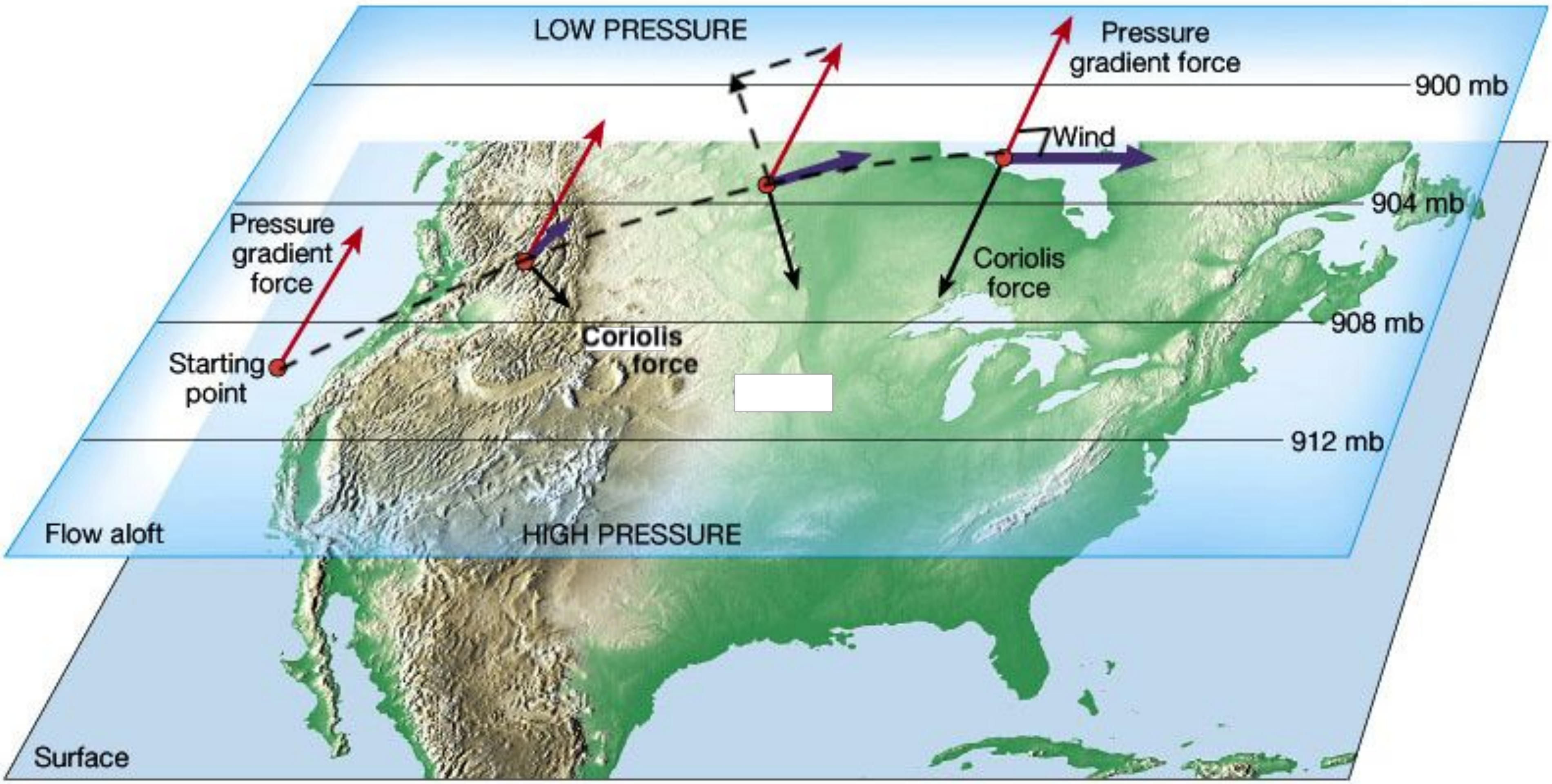



**MERIDIONAL CIRCULATION AND DISTORTION OF A SIMPLE HADLEY CELL**

# ACTUAL CELL AIR FLOW (2)



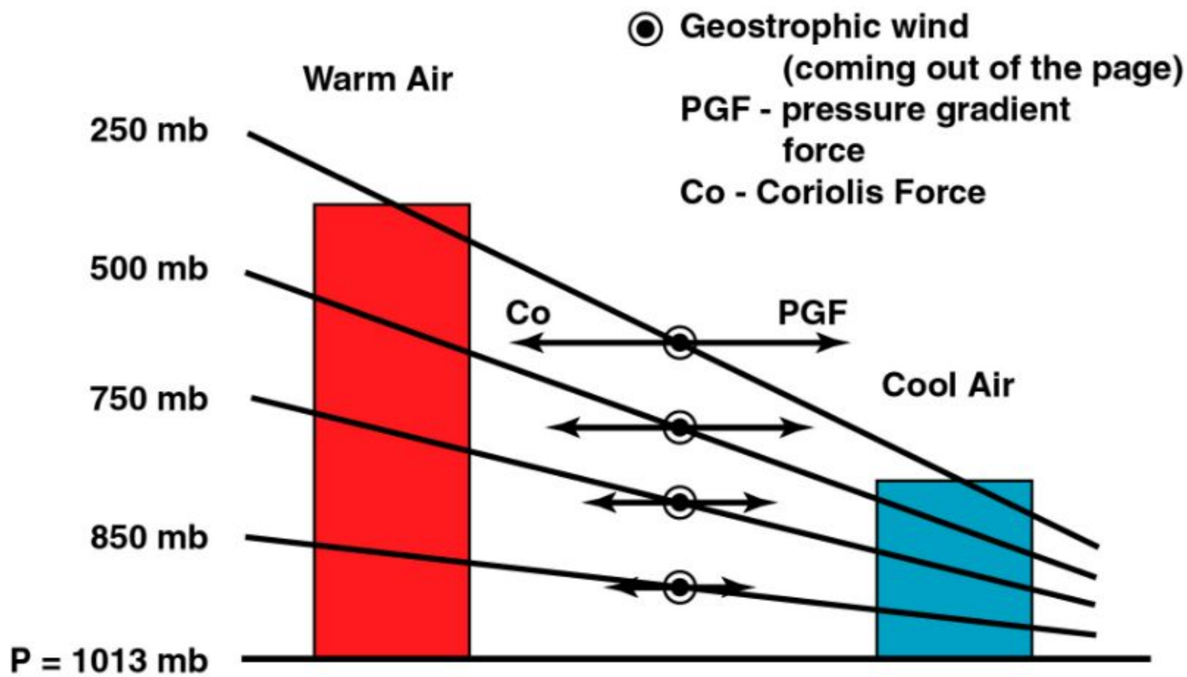
# GEOSTROPHIC WIND




**THE PRESSURE GRADIENT FORCE EXACTLY BALANCES THE CORIOLIS FORCE**
**THE GEOSTROPHIC WIND DOMINATES AT HIGH ALTITUDE**

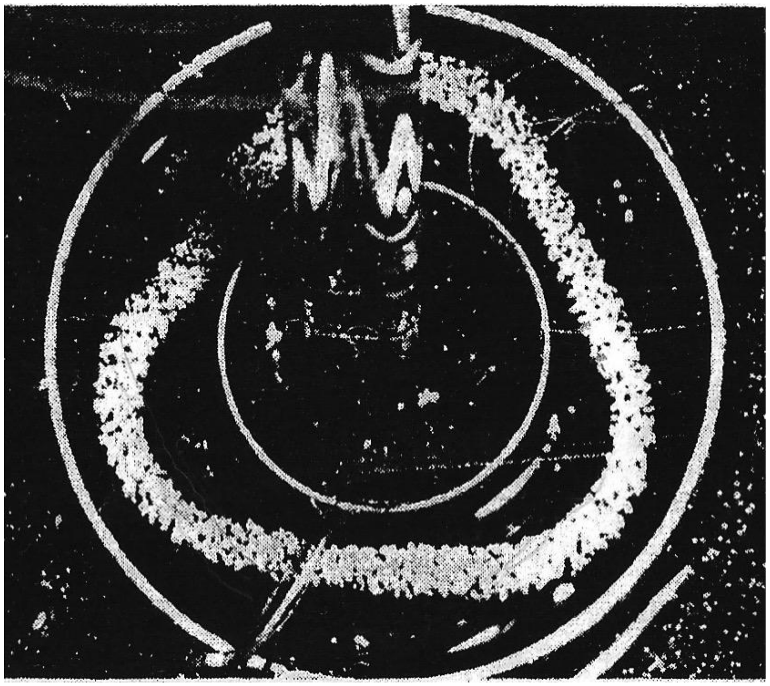
# JET STREAMS

- **BALANCE OF THE PRESSURE GRADIENT AND CORIOLIS FORCES WITH ALTITUDE:**

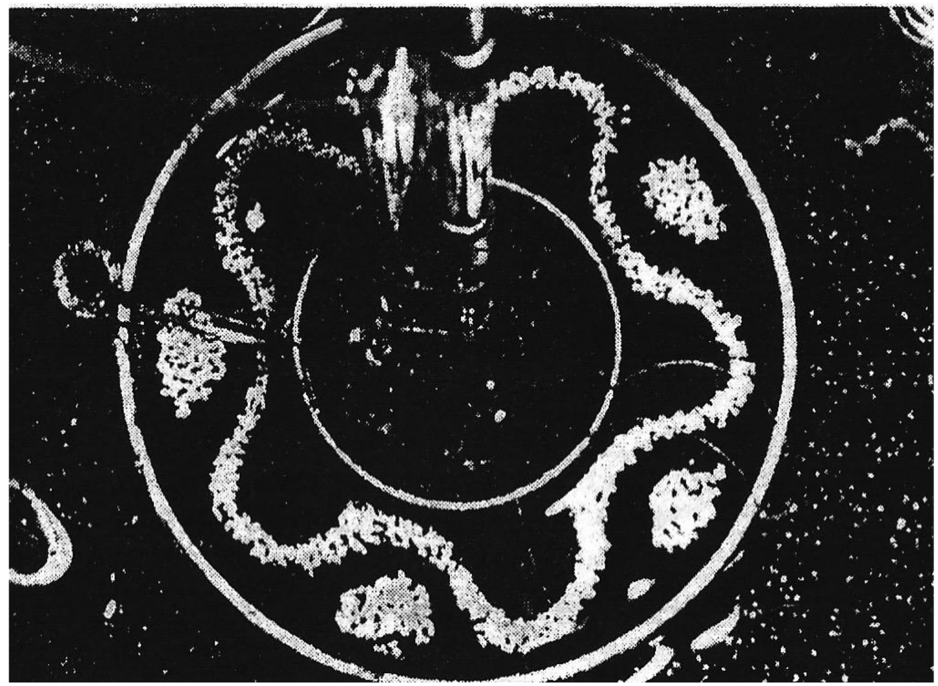



**THE STRONG WIND AT HIGH ALTITUDE IS CALLED THE JET STREAM**

# FLUID-DYNAMICAL ANALOGUES OF ROSSSBY WAVES



3-**WAVE PATTERN**



**5-WAVE PATTERN**

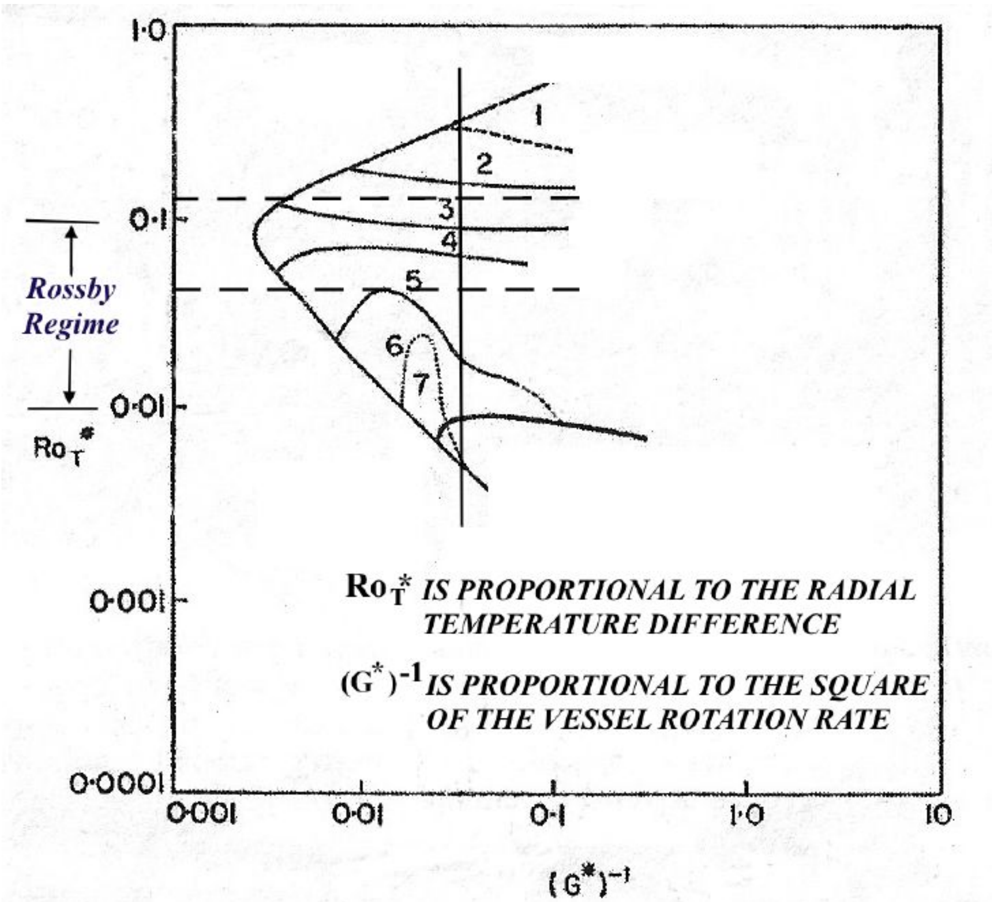
**ALUMINUM DUST ON ROTATING ANULUS OF WATER. INNER CYLINDER COOLED AND OUTER HEATED MAINTAINING CONSTANT MEAN TEMPERATURE. 3-WAVE THERMAL ROSSBY NUMBER 0.12; 5-WAVE 0.058 [THERMAL ROSSBY NUMBER FUNCTIONAL DEPENDENCE** $\sim \Delta T/\Omega^2$ ] **(D. FULTZ-MID 1950s. A LECTURE BY DAVE FULTZ ON ROTATING FLOWS IS AT: https://www.youtube.com/watch?v=Ans3tnvMyTk)**

# FULTZ LAB EXPERIMENTS

- **A CRITICAL POINT WAS FOUND BY FULTZ WHEN THE TEMPERATURE DIFFERENCE BETWEEN THE TWO CYLINDERS WAS RAISED (MAINTAINING CONSTANT MEAN TEMPERATURE)**

- **AT THE CRITICAL POINT A 7-WAVE PATTERN WAS FORMED FOLLOWED SEQUENTIALLY BY LOWER WAVE NUMBER PATTERNS**

# SIMPLIFIED TRANSITION CURVES




**THE HEAVY BULLET SHAPED CURVE SEPARATES THE HADLEY AND WAVY ROSSBY REGIMES**

**THE VARIABLES** $Ro_T^*$ **(THE THERMAL ROSSBY NUMBER) AND** $(G*)^{-1}$ **ARE DIMENSIONLESS SO AS TO MAKE THE RESULTS SCALE INVARIANT. THE VERTICAL LINE CORRESPONDS TO THE ROTATION RATE IN THE PHOTOS SHOWN ABOVE. FOR SMALL TEMPERATURE DIFFERENCES, AS IS THE CASE FOR SATURN, ONE WOULD EXPECT A SIX WAVE PATTERN.**

# SIMPLIFIED TRANSITION CURVES (2)

- MORE DETAILED TRANSITION CURVES DISTINGUISH BETWEEN WHETHER THE CORE OR RIM IN THE EXPERIMENTS IS HEATED.
- CURRENTS AND JETS AT THE TOP OF THE FLUID FLOW:
  - EASTWARD IF THE RIM IS HEATED AND THE CORE COOLED.
  - WESTWARD IF THE CORE IS HEATED AND THE RIM COOLED.
- APPARENTLY INCONSISTENT WITH SATURN JET WHERE POLAR SIDE IS WARMER THAN MERIDIONAL SIDE AND JET FLOWS TO THE EAST. BUT TEMPERATURE RISE ON POLAR SIDE APPEARS TO BEGIN WITH THE HEXAGONAL STRUCTURE AT 79 $^{o}$N. (REMEMBER THE GAP BETWEEN THE HEXAGON JET AND THE HEXAGONAL STRUCTURE.)

# SCALABILITY TO SATURN

$$(G^*)^{-1} \equiv \frac{r_0 \Omega^2}{g}$$

$r_0$ = Reference radius
● = Angular velocity of rotation
$g$ = Gravitational acceleration

**FOR SATURN:**
$r_0$ = Radius of hexagon at 1 bar = $1.5 \times 10^7$ m
$\Omega = 1.6 \times 10^{-4}$ rad/sec
$g$ = 10.44 m/sec$^2$

**Results in $(G^*)^{-1}$ = 0.036 implying that the Fultz transition curves may be scalable to Saturn if the thermal Rossby number is compatible with a wave number of 6.**

# SCALABILITY TO SATURN (2)

## ROSSBY NUMBER FOR FULTZ's ANNULAR CONFIGURATION

$$R_0 = U/[\Omega(b-a)]$$

**Ω is the angular velocity of the tank; *U* is the relative velocity of the fluid; and (*b* – *a*) is the difference in radius between the outer wall and the inner wall of the annular region. If *h* is the depth of the fluid; Δ*T* the radial temperature difference across the annulus; ε the thermal expansion coefficient, *U* can be approximated as**

$$U \sim \varepsilon g h\,(\Delta T)/[2\,\Omega(b-a)]$$

**Putting this into the Rossby number above gives the THERMAL ROSSBY NUMBER**

$$R_{0_T} = \varepsilon g h\,(\Delta T)/[2\,\Omega^2(b-a)^2]$$

**This differs from the form used by Fultz, but works for his experiments.**

# SCALABILITY TO SATURN (3)

- **FOR SATURN:**
- **(b−a) = width of the hexagon = 6 $\times$ $10^6$ m**
  **$g$ = 10.44 m/sec$^2$; $\Delta T$ = 1 K; $\Omega$ = 1.6 $\times$ $10^{-4}$ rad/sec;**
  **$\varepsilon$ = 3.66 $\times$ $10^{-3}$ (Change in volume at constant pressure at 0 $^o$C per unit volume per $^o$C)**
  ***h* Is not known, but there is evidence to indicate that the hexagon does not penetrate deep into the atmosphere.**

- **FOR THE HEXAGON TO HAVE A WAVE NUMBER OF 6, GIVEN THAT $(G^*)^{-1}$ = 0.036, THE THERMAL ROSSBY NUMBER MUST BE ~0.05. USE THIS TO CALCULATE *h*. THE RESULT IS *h* = 2.5 $\times$ $10^6$m. (THE TEMPERATURE AT THIS DEPTH IS ~1700 $^o$K)**

- **THE FULTZ TRANSITION CURVES APPEAR TO BE SCALABLE TO THE DIMENSIONS OF SATURN!**

# MORE RECENT EXPERIMENTS

- **BARBOSA AGUIAR, ET AL. (2010)**

  **-REGULAR STABLE POLYGONS CAN FORM WITH ASSOCIATED TRAIN OF VORTICES—BOUNDING CYLINDERS NOT HEATED OR COOLED**

  **-ZONAL WAVE NUMBER [$k := (2\pi r \cos\varphi)/\lambda$] DEPENDS ON WAVELENGTH THAT IS "ENERGETICALLY FAVORED"**

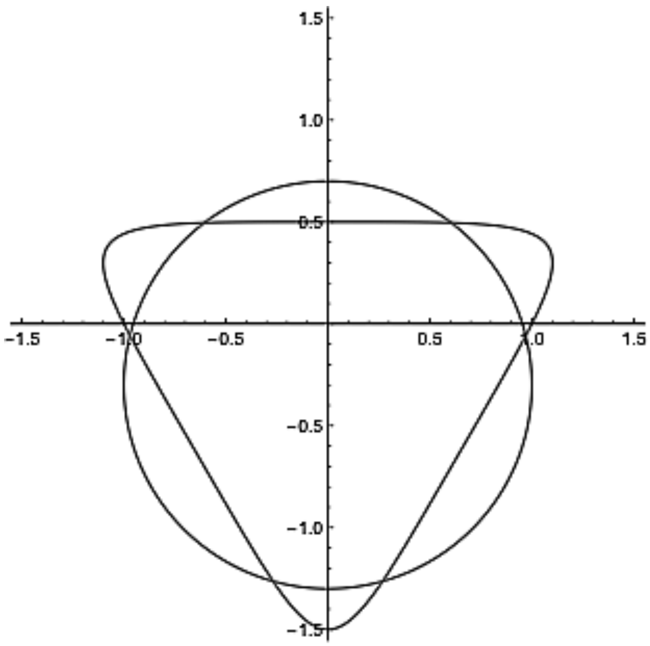
- **MORALES-JUBERIAS, ET AL. (2015)**

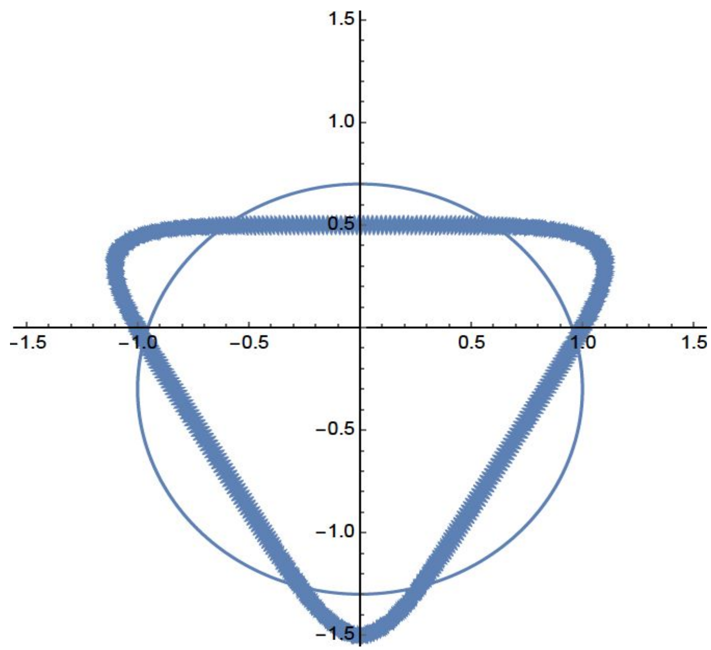
  **-USED A FLUID DYNAMICAL MODEL TO SIMULATE SATURN'S HEXAGON**

  **-PEAK VELOCITY OF THE JET DETERMINED THE DOMINANT WAVE NUMBER**

- **(REFERENCES AVAILABLE FROM MY ARTICLE ON WEBSITE gemarsh.com OR FROM [https://arxiv.org/pdf/1711.00338](https://arxiv.org/pdf/1711.00338))**

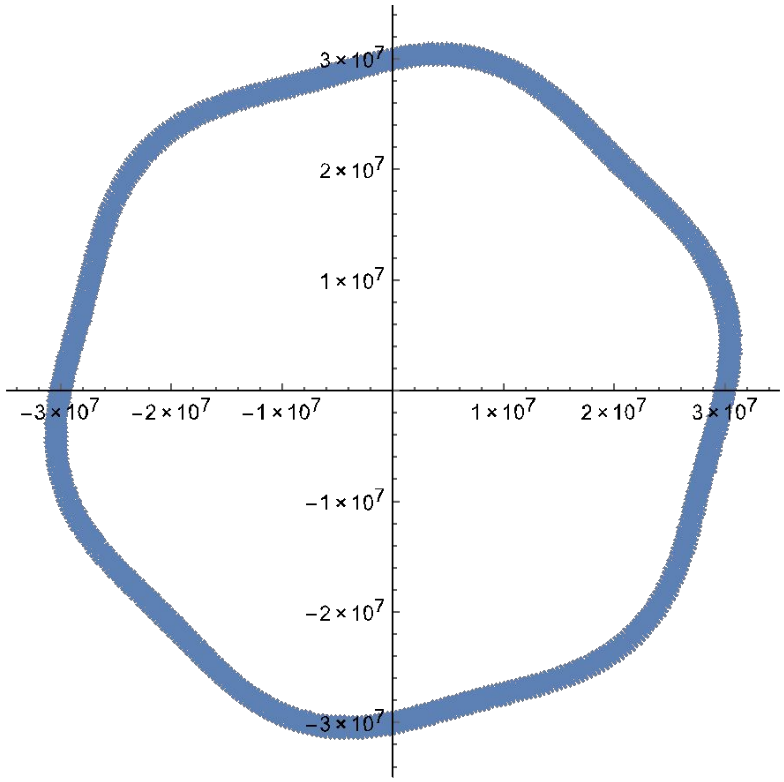
# WAVES ON A CIRCLE
# ANALYTIC VERSION OF FULTZ'S 3-LOBED ROSSBY WAVE




- **THE CIRCLE HAS ITS ORIGIN AT THE CENTROID OR CENTER OF GRAVITY OF THE TRIANGLE—A PARAMETRIC PLOT OF (1 + 0.5 SIN$\theta$ ) FROM 0 TO 2$\pi$**
- **THE SAME OFFSET CAN BE SEEN IN THE FULTZ-LAB 3-WAVE PHOTO**
- **THE PLOT ON THE RIGHT HAS A SHORTER WAVELENGTH ADDED.**

# ANALYTIC VERSION OF SATURN'S HEXAGON




- HEXAGON OBTAINED USING THE PARAMETERS FOR SATURN'S JET
- A SHORTER WAVELENGTH HAS ALSO BEEN ADDED HERE WITH ITS AMPLITUDE SET TO THE WIDTH OF THE JET
- THIS SHOWS THAT—HEURISTICALLY—THE CONFIGURATION OF SATURN'S JET CAN BE OBTAINED BY SIMPLY ADDING TWO DIFFERENT FREQUENCY WAVES ON A CIRCLE.

**<u>BUT THE WAVES OF INTEREST ARE COMBINATIONS OF ROSSBY WAVES OF VARYING WAVELENGTH!</u>**

# DEFINITIONS AND ASSUMPTIONS

## VORTICITY IS THE CURL OF THE VELOCITY

- **ATMOSPHERE IS TREATED AS HOMOGENEOUS, INCOMPRESSIBLE, INVISCID, HAVING ONLY HORIZONTAL MOTIONS.**

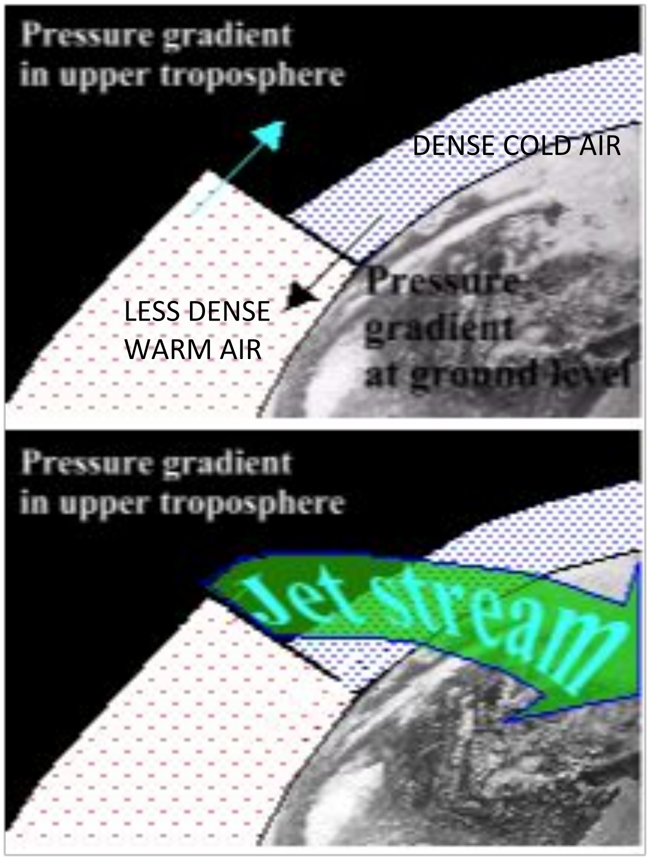
- **IF THE COMPONENTS OF THE VELOCITY *U* ARE (*u*, *v*, *w*), THE RELATIVE VORTICITY IS $\varsigma_r = (\partial_x v - \partial_y u)$, WHERE *THE* SUBSCRIPT *r* MEANS RELATIVE TO THE EARTH.**

- **POTENTIAL VORTICITY OF A THIN FLUID LAYER IS $(\zeta_r + f)/h$, WHERE $f = 2\omega \sin\varphi$ IS THE CORIOLIS PARAMETER, AND *h* IS THE THICKNESS OF THE LAYER.**

- **ABSOLUTE VORTICITY IS $\zeta_a = \zeta_r + f$ , WHERE THE SUBSCRIPT *a* MEANS RELATIVE TO ABSOLUTE SPACE.**

- **THE $\beta$-PLANE APPROXIMATION IS $f = f_0 + \beta y$, WHERE $f_0$ IS THE CORIOLIS PARAMETER AT A PARTICULAR LATITUDE, $\beta = \partial_y f$ , AND *y* IS THE COORDINATE TANGENT TO THE SURFACE OF THE EARTH POINTING TO THE NORTH.**

# MORE ON JETS AND ROSSBY WAVES




- **COLUMNS OF AIR THAT INCREASE THEIR RADIUS DECREASE BOTH THEIR VERTICAL EXTENT AND ABSOLUTE VORTICITY AND THOSE WHICH DECREASE THEIR RADIUS INCREASE BOTH**
- **THERE IS ALARGE LATERAL GRADIENT OF POTENTIAL VORTICITY PV = $(\zeta_r+f)/h$ ACROSS THE STEP**
- **LARGE LATERAL POTENTIAL VORTICITY GRADIENT AND THE INFLUENCE OF THE CORIOLIS FORCE RESULT IN THE JET STREAM**
- **MEANDERS IN THE JET STREAM ARE ARE CALLED ROSSBY WAVES. ON EARTH MEANDERS CAN BE LARGE DUE TO SURFACE TOPOGRAPHY. SATURN IS A GAS GIANT WITH NO SURFACE AS WE KNOW IT SO MEANDERS CAN BE EXPECTED TO BE MINIMAL**

**ROSSBY WAVES ARE TRANSVERSE WAVES WHOSE RESTORING FORCE UNDER NORTH OR SOUTH DISPLACEMENT IS PROPORTIONAL TO THE CHANGE IN THE CORIOLIS PARAMETER**

# ROSSBY WAVES

- **BAROTROPIC (DENSITY ONLY A FUNCTION OF PRESSURE) RELATIVE VORTICITY EQUATION**

$$\frac{\partial \varsigma_r}{\partial t} + u\frac{\partial \varsigma_r}{\partial x} + v\frac{\partial \varsigma_r}{\partial y} + \beta v = 0$$

**WHERE $\varsigma_r = (\partial_x v - \partial_y u)$.**

**THE SATURN JET DOESN'T DO MUCH MEANDERING SO THAT $\partial_x v = \partial_t \varsigma_r \approx 0$ AND RELABELING $u$ AS $U$ GIVES AN EXPRESSION FOR $\beta$**

$$\beta = \frac{\partial^2 U}{\partial y^2}$$

**THE R.H.S. IS CALLED THE RELATIVE VORTICITY GRADIENT.**

# ROSSBY WAVE DISPERSION RELATION

- **FOR AN INCOMPRESSIBLE AND HORIZONTALLY HOMOGENEOUS ATMOSPHERE THE PHASE VELOCITY OF ROSSBY WAVES CAN BE SHOWN TO BE**

$$c = U - \frac{\beta \lambda^2}{4\pi^2}$$

**STATIONARY WAVES ARE GIVEN BY SETTING *c* = 0, RESULTING IN THE DISPERSION RELATION (SUBSCRIPT *s* MEANS STATIONARY)**

$$\lambda_s = 2\pi \left(\frac{U}{\beta}\right)^{1/2}$$

# ROSSBY WAVE DISPERSION RELATION

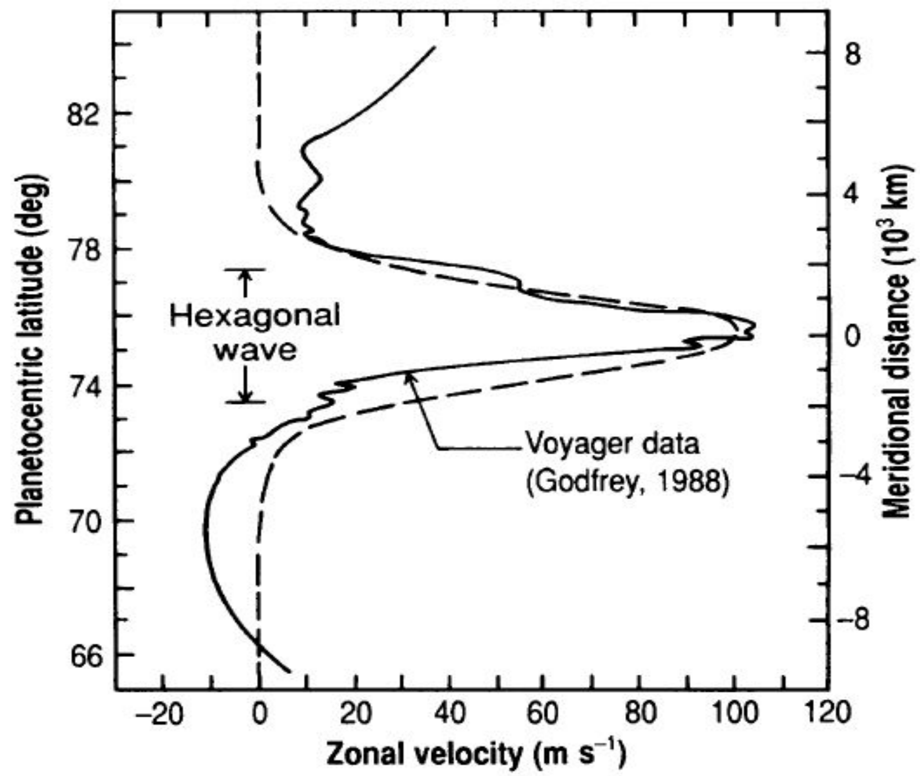
- SUBSTITUTING THE PREVIOUS RELATION FOR $\beta$ INTO THIS RELATION GIVES

$$\lambda_s = 2\pi U^{1/2}\left(\frac{\partial^2 U}{\partial y^2}\right)^{-1/2}$$

| AUTHOR | RELATIVE VORTICITY GRADIENT ($m^{-1}\ s^{-1}$) | $\lambda_s$ (m) | $n = C / \lambda_s$ |
|---|---|---|---|
| ALLISON, ET AL. | $2.2 \times 10^{-11}$ | $1.3 \times 10^{7}$ | 7.2 |
| DEL GENIO, ET AL. | $3.0 \times 10^{-11}$ | $1.2 \times 10^{7}$ | 7.8 |
| AGUIAR, ET AL. | $0.7 \times 10^{-11}$ | $2.37 \times 10^{7}$ | 3.9 |

**AVERAGING THESE RELATIVE VORTICITY GRADIENTS RESULTS IN *n* = 5.8.**
**(NOTE THAT *n* FOR SATURN IS 6. THE VELOCITY *U* USED FOR THE TABLE IS 100 m/s AND *C* IS THE CIRCUMFERENCEOF THE LATITUDE CIRCLE AT 76$^{\circ}$ NORTH, THE CENTRAL POSITION OF THE JET.)**

# VELOCITY PROFILE FOR THE HEXAGONAL JET




ZONAL (LATITUDINAL) VELOCITY PROFILE FOR THE HEXAGONAL JET. THE DASHED LINE IS THE GAUSSIAN CURVE USED TO APPROXIMATE THE VOYAGER DATA. NOTE THAT THE GAUSSIAN PROFILE SATISFIES THE BOUNDARY CONDITION *U* = 0 AT THE EDGES OF THE JET. THE FLOW IS TO THE EAST. (ADAPTED FROM ALLISON, ET AL.)

# JET VELOCITY PROFILE

**THE GAUSSIAN APPROXIMATION TO THE JET'S VELOCITY PROFILE IS**

$$U = U_0 e^{-by^2/2U_0}$$

***b*** **CAN BE FOUND BY TAKING THE SECOND DERIVATIVE AND EVALUATING IT AT *y* = 0**

$$b = \left| -\frac{\partial^2 U}{\partial y^2} \right|_{y=0}$$

**THIS ALLOWS THE USE OF THE PREVIOUS EXPRESSION FOR THE STATIONARY WAVELENGTH.**

# JET VELOCITY PROFILE (2)

•**THE LATITUDINAL CERCUMFERENCE AT 76$^{O}$ N WHERE THE HEXAGON IS LOCATED IS 9.4 $\times$ 10$^{7}$ m. THE WAVELENGTH FOR *n* = 6 IS THEN 1.56 $\times$ 10$^{7}$ m. USING THE PEAK VELOCITY OF 100 *m/s* IN**

$$\lambda_s = 2\pi U^{1/2}\left(\frac{\partial^2 U}{\partial y^2}\right)^{-1/2}$$

**RESULTS IN *b* = 1.6 $\times$ 10$^{-11}$ m$^{-1}$ s$^{-1}$—WITHIN THE RANGE SHOWN EARLIER FOR THE RELATIVE VORTICITY GRADIENT GIVEN BY DIFFERENT AUTHORS.**

•**THE KEY ISSUE IS HOW TO PUT TOGETHER THE MULTIPLE ROSSBY WAVES OF VARYING WAVELENGTHS THAT RESULT FROM THE VELOCITY PROFILE.**

# ROSSBY WAVES REDUX

- **ROSSBY WAVES TRAVELING IN AN ARBITRARY DIRECTION—TWO COMPONENT WAVE VECTORS**

**DEFINE A STREAM FUNCTION *ψ* FOR SATURN'S HEXAGON SUCH THAT** $u = -\partial_y \psi$ **AND** $v = \partial_x \psi$**, WHERE THE VELOCITY *U* = (*u*, *v*). THE VORTICITY IS THE *z*-COMPONENT OF THE RELATIVE VORTICITY** $\varsigma_r$**: IN TERMS OF THE STREAM FUNCTION,** $\varsigma_r = \nabla^2 \psi$**, WHERE** $\nabla^2$ **IS THE 2-DIMENSIONAL LAPLACIAN. THE BAROTROPIC VORTICITY EQUATION IN A FRAME FOLLOWING THE MOTION BECOMES:**

$$\frac{\partial}{\partial t}\nabla^2\psi + \beta\,\frac{\partial \psi}{\partial x} = 0$$

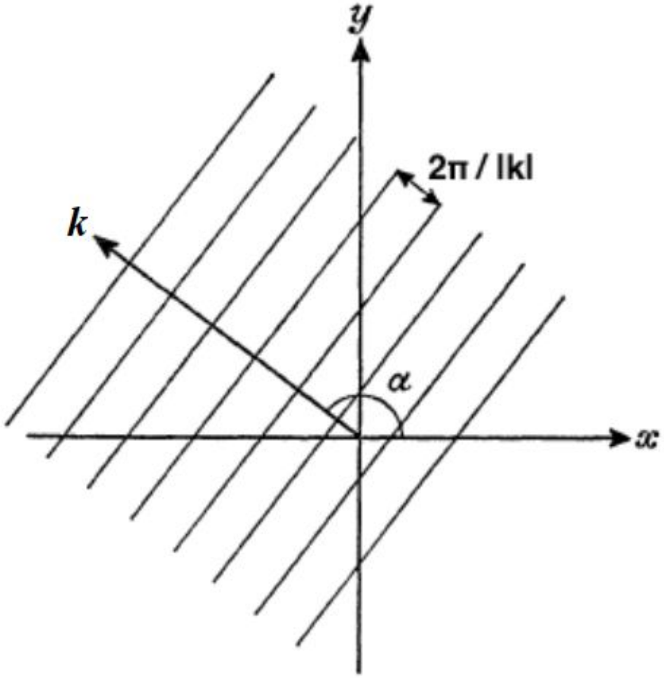
**LET** $\psi = \exp[i(\vec{k} \bullet \vec{x} - \omega t)]$ **, WHERE** $\vec{k} = (k, l)$ **AND** $\vec{x} = (x, y)$

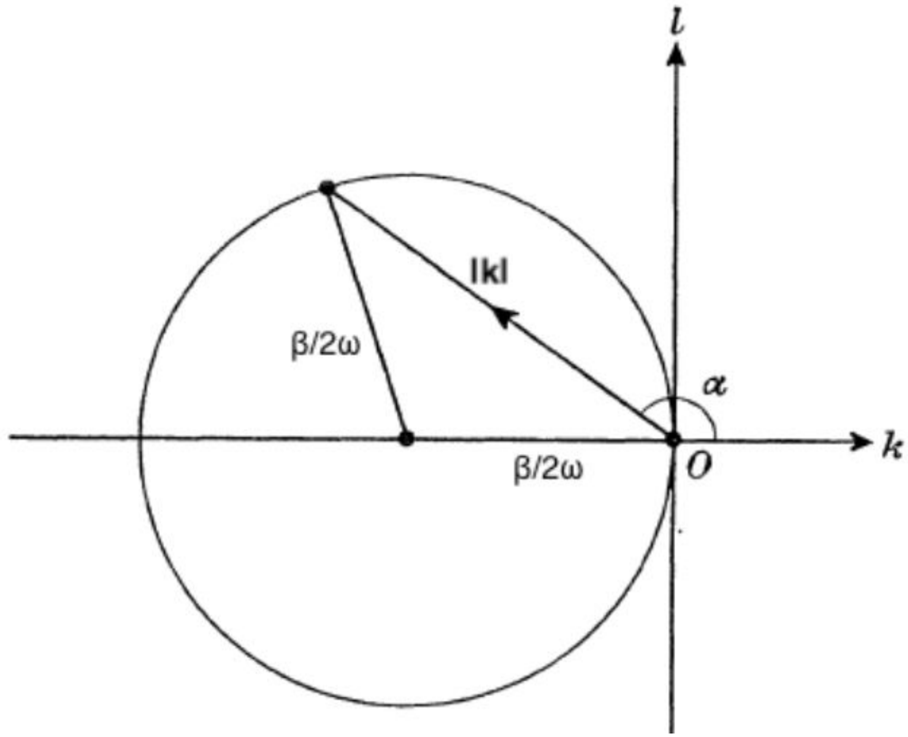
**THIS RESULTS IN AN EXPRESSION RELATING THE FREQUENCY *ω* AND** $|\vec{k}|$

# FREQUENCY AND WAVE NUMBER

$$\frac{\omega}{k} = -\frac{\beta}{k^2 + l^2} \qquad \omega = -\frac{\beta \cos \alpha}{|\vec{k}|}$$

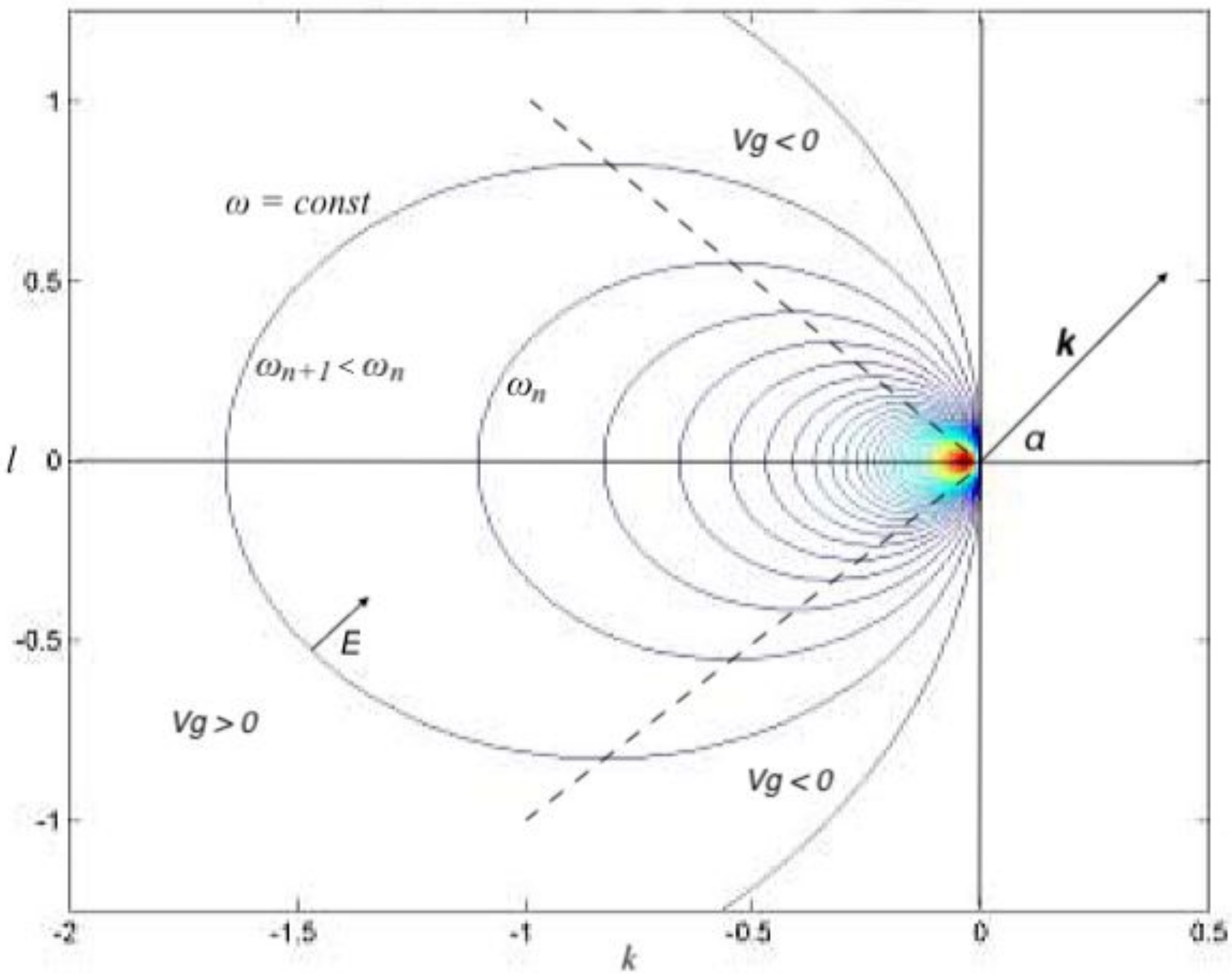
**THE MINUS SIGN MEANS THE PHASE PLANES HAVE A WESTWARD VELOCITY COMPONENT. NOTE THAT $\omega/k$ IS THE PHASE VELOCITY.**




**($k$, $l$ ) = (|$k$|cos$\alpha$, |$k$|sin$\alpha$)   SURFACES OF CONSTANT $\omega$ ARE CIRCLES**

# FREQUENCY AND WAVE NUMBER (2)




**(NOTE ASPECT RATIO: SURFACES OF CONSTANT PHASE ARE ACTUALLY CIRCLES)**

# PHASE & GROUP VELOCITY

- $\omega/k$ IS THE PHASE VELOCITY
- THE GROUP VELOCITY IS THE GRADIENT OF $\omega$:

$$\nabla\omega = \beta\,\frac{\cos 2\alpha}{\left|\vec{k}\right|}\;\hat{k} + 2\beta\,\frac{kl}{\left|\vec{k}\right|^{4}}\;\hat{l}$$

**THE GROUP VELOCITY VECTOR MAKES AN ANGLE 2$\alpha$ WITH RESPECT TO THE *X*-AXIS AND POINTS INWARD TO THE CENTER OF THE CIRCLES FROM POINTS ON THE SURFACES OF CONSTANT $\omega$ . IT IS NORMAL TO THESE SURFACES.**

**NOTE THAT WHEN *Vg* > 0 – (NORMAL DISPERSION) – THE FLOW IS TO THE EAST OPPOSITE TO THE DIRECTION OF THE PHASE VELOCITY.**

# GROUP VELOCITY

$$\omega = -\frac{\beta \cos \alpha}{|\vec{k}|} \qquad V_g = \frac{\partial \omega}{\partial k} = \frac{1}{k^2} \frac{\left(1 - \frac{l^2}{k^2}\right)}{\left(1 + \frac{l^2}{k^2}\right)^2}.$$

- **FOR *l* > *k* THE GROUP VELOCITY IS NEGATIVE *APPARENTLY* ALLOWING THE VIOLATION OF CAUSALITY. BUT SIGNAL AND GROUP VELOCITY ONLY COINCIDE IN REGIONS OF NORMAL DISPERSION. THIS IS NOT THE CASE IN REGIONS OF ANOMALOUS DISPERSION, WHERE *l* > *k.***

- **LASER EXPERIMENTS SHOW THAT WHILE THE PEAK OF A PULSE IN A REGION OF ANOMALOUS DISPERSION CAN PROPAGATE BACKWARDS, THE ENERGY FLOW IS ALWAYS IN THE FORWARD DIRECTION. CAUSALITY IS NOT VIOLATED.**

# COMBINING ROSSBY WAVES

## GEORGE PLATZMAN (1968 SYMONS MEMORIAL LECTURE) INTRODUCED A WAVEGUIDE ANALOGY FOR ROSSBY WAVES

**THE VORTICITY EQUATION IS**

$$\frac{d}{dt}\nabla^2\psi + 2\Omega\frac{\partial\psi}{\partial\lambda} = 0$$

**$\psi$ IS THE STREAM FUNCTION, $\nabla^2$ THE SURFACE SPHERICAL LAPLACIAN, $\Omega$ THE EARTH'S ROTATIONAL SPEED AND $\lambda$ THE GEOGRAPHICAL LATITUDE. THE BETA PLANE CORRESPONDENCE FOLLOWS FROM $\nabla^2\psi$ BEING THE VORTICITY AND $2\Omega\ \partial\psi/\partial\lambda$ BEING THE SAME AS $\beta v$.**

# COMBINING ROSSBY WAVES (2)

**IF THE INSTANTANEOUS DISTRIBUTION OF $\psi$ OVER THE SPHERE CORRESPONDS TO A SPHERICAL SURFACE HARMONIC SO THAT AT THAT INSTANT $\psi$ SATISFIES $\nabla^2\psi + n(n+1)\psi = 0$, THE VORTICITY EQUATION BECOMES**

$$\frac{d\psi}{dt} - \frac{2\Omega}{n(n+1)}\frac{\partial\psi}{\partial\lambda} = 0$$

**THE RESULTING SPHERICAL HARMONIC PATTERN WILL DRIFT WESTWARD WITHOUT CHANGE OF SHAPE AT AN ANGULAR SPEED OF $2\Omega/n(n+1)$ IN GEOGRAPHIC LONGITUDE (ANALOGOUS TO THE PHASE VELOCITY ABOVE). NOTE THAT INTEGRAL VALUES OF *n* ARE THE ONLY ZONAL SURFACE HARMONICS WHICH ARE FINITE OVER THE UNIT SPHERE.**

**FOR SATURN'S HEXAGON, $n = 1$ SO THAT THE HEXAGON IS STATIONARY WITH RESPECT TO ABSOLUTE SPACE.**

# WAVEGUIDES

**FOR *n* AN INTEGER, THE SOLUTION CAN BE REPRESENTED AS A SUM OF TESSERAL HARMONICS. A TESSERAL HARMONIC HAS THE FORM OF cos($m\Phi$) OR sin($m\Phi$) TIMES A LEGENDRE POLYNOMIAL $P_l^m(\cos\vartheta)$. THEY VANISH ON ($l - m$) PARALLELS OF LATITUDE AND 2*m* MERIDIANS, DIVIDING THE SURFACE OF A SPHERE INTO QUADRANGLES WHOSE ANGLES ARE RIGHT ANGLES.**

TYPES OF SPHERICAL HARMONICS

**ZONAL HARMONICS ARE GIVEN BY $m = 0$ AND SECTORAL BY $m = n$**

**PLATZMAN NOTED THAT THE NODAL CONFIGURATION OF TESSERAL HARMONICS IS SIMILAR TO THAT OF A WAVE-GUIDE MODE AND INTRODUCED THAT TERM.**

# WAVEGUIDES (2)

**TO GET A SOLUTION THAT IS THE ANALOGUE OF A ROSSBY WAVE LONGUET-HIGGINS SHOWED IN 1964 THAT ONE CAN USE A SUM OF WAVES INVOLVING ASSOCIATED LEGENDRE FUNCTIONS OF THE FIRST AND SECOND KIND. SUCH A SUM CAN RESULT IN A SECTORAL WAVE-GUIDE MODE APPLICABLE TO ROSSBY WAVES.**

**FOR WAVES CONSTRAINED BY TWO LATITUDES (WHICH WOULD BE APPLICABLE TO SATURN'S HEXAGON) HE FOUND THAT THE SOLUTION MUST BE IN THE FORM**

$$\psi = [AP_n^s(\cos\theta) + BQ_n^s(\cos\theta)]e^{is(\emptyset - \dot{\emptyset}t)}$$

**WHERE *n* IS *NOT* GENERALLY INTEGRAL.**

# WAVEGUIDES (3)

- **FOR THE HEXAGON, *n* = 1, AND FOR A SECTORAL MODE, ONLY $P_1^1$ D AR$Q_1^1$ VAILABLE. IF *A* AND *B* ARE CONSTANTS, IT DOES NOT APPEAR TO BE POSSIBLE TO SATISFY THE BOUNDARY CONDITION THAT $\psi$ VANISH ON THE TWO CONSTRAINING LATITUDES WITH ONLY THESE FUNCTIONS.**

- **NONETHELESS , IN THE GENERAL CASE, SOLUTIONS OVER A SPHERE MAY BE FOUND THAT CORRESPOND LOCALLY TO WAVES ON A $\beta$-PLANE TRAPPED BY THE CURVATURE OF THE ROTATING SPHERE. THIS TRAPPING EFFECTIVELY COMBINES WAVES OF DIFFERING WAVELENGTH.**

# WAVEGUIDES (4)

- IT WAS SHOWN EARLIER THAT TROPOPAUSE LEVEL JET STREAMS HAVE A STEPLIKE DECREASE IN THE HEIGHT OF THE TROPOPAUSE AND A STRONG LATERAL POTENTIAL VORTICITY GRADIENT. THIS DISCONTINUITY CAN ALSO ACT AS A "WAVEGUIDE" TRAPING SMALL-AMPLITUDE ROSSBY WAVES THAT THEN PROPAGATE ALONG THE DISCONTINUITY.

- SINCE IT DOES NOT APPEAR TO BE POSSIBLE TO USE THE LONGUET-HIGGINS SOLUTION FOR SATURN'S HEXAGON, THIS MECHANISM—OR SOMETHING LIKE IT—IS PROBABLY RESPONSIBLE FOR COMBINING DIFFERENT WAVELENGTH ROSSBY WAVES.

- IN THE LITERATURE, THE TERMS JET AND WAVEGUIDE ARE OFTEN USED INTERCHANGEABLY.

# SUMMARY

- WHILE MANY QUESTIONS REMAIN ABOUT SATURN'S HEXAGON, ITS EXISTANCE IS NO LONGER AS ENIGMATIC AS IT WAS WHEN IT WAS FIRST DISCOVERED.

- SATURN'S HEXAGON IS PERHAPS THE ONLY CASE OF A DIRECTLY OBSERVABLE ROSSBY WAVE. THE CONTRAST ALLOWING IT TO BE VISIBLE IS DUE TO A DIFFERENCE IN THE SIZE OF THE LIGHT SCATTERING PARTICLES WITHIN THE HEXAGON AND OUTSIDE IT.

# SUMMARY (2)

- UNLIKE EARTH, SATURN IS A GAS GIANT WITH NO SURFACE AS WE KNOW IT SO MEANDERS IN A JET WOULD BE EXPECTED TO BE MINIMAL ALLOWING A JET WITH A STABLE POLYGONAL SHAPE TO FORM.

- FULTZ'S TRANSITION CURVES PREDICT THAT FOR A SMALL TEMPERATURE GRADIENT THE ROSSBY WAVE SHOULD TAKE THE FORM OF A HEXAGON—AND THIS IS WHAT WE SEE ON SATURN.

# SUMMARY (3)

- FULTZ WANTED HIS RESULTS TO BE SCALE INVARIANT BUT COULD NOT POSSIBLY HAVE IMAGINED THAT SIXTY YEARS LATER THEY WOULD BE SCALED TO THE SIZE OF SATURN!

- THE ACTUAL NATURE OF THE INTERACTION AND CHANNELING OF THE ROSSBY WAVES COMPRISING THE HEXAGON IS STILL OPEN AS IS A DERIVATION OF THE VELOCITY PROFILE.


# THANK YOU!