Topology and the Baryon-Antibaryon Asymmetry in the Early Universe

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ABSTRACT

It is currently believed that baryon number conservation must be violated to produce a preponderance of baryons over anti-baryons in the early universe. An alternative to violating baryon number conservation is given here that preserves CPT invariance.

Introduction

The baryon-antibaryon asymmetry in the early universe cannot be explained within the Standard Model of particle physics and cosmology. The idea of breaking baryon number symmetry between baryons and anti-baryons—resulting in *C* and *CP* invariance being violated—so as to obtain a preponderance of matter over antimatter is fraught with problems. Another approach is needed. One is offered below. It involves topological change, which has usually been ruled out, but the arguments against it have been found to be wanting. This is discussed in some detail below.

The FRW metric and 3-Spheres

Seconds after the Big Bang the universe can be accurately described by an approximately spatially-flat, radiation-dominated FRW metric. The designation FRW comes from Robinson and Walker finding that the Friedman-Lemaître metric can be put into the form

$$ds^2 = -dt^2 + S^2(t)d\sigma^2,$$

where, in spherical coordinates $d\sigma^2$ is given by

$$d\sigma^2 = \left(d\chi^2 + f^2(\chi)(d\theta^2 + \chi^2(d\theta^2 + sin^2\theta\ d\phi^2)\right).$$

Eqs. (1)

The metric $d\sigma^2$ is a 3-dimensional hypersurface of constant curvature independent of time. The second of Eqs. (1) can have three values for $f(\chi)$ depending on the normalized curvature *K* of the universe obtained by rescaling the function *S*:

$$f(\chi) = \begin{pmatrix} sin\chi\ if\ K = +1 \\ \chi\ if\ K = 0 \\ sinh\chi\ if\ K = -1 \end{pmatrix}.$$

Eq. (2)

If K = 0 or −1, the 3-dimensional hypersurfaces are diffeomorphic to three-dimensional flat spaces and if K = +1 it is diffeomorphic to the three-sphere $\mathbb{S}^3$. For K = 0 or −1, the range of χ is $0 \leq \chi \leq \infty$, and for K = +1, the range is $0 \leq \chi \leq 2\pi$.

Current astronomical observations and measurements constrain the spatial curvature of the universe to be very close to zero, but cannot determine the sign of the curvature if it exists. The case of a very slight positive curvature will be considered here, in which case the second of Eqs. (1) describes the spatial geometry of a 3-Sphere $\mathbb{S}^3$. A 3-Sphere is a compact manifold and while being finite does not have a boundary. This choice is made because it offers a possible explanation for the preponderance of baryons over anti-baryons in the early universe.

The 3-Sphere

The 3-Sphere can be seen to be the union of two 3-Balls by a homeomorphism[1] h, which maps the boundary of B_1^3 onto the boundary of B_2^3. This is illustrated in Fig. 1.

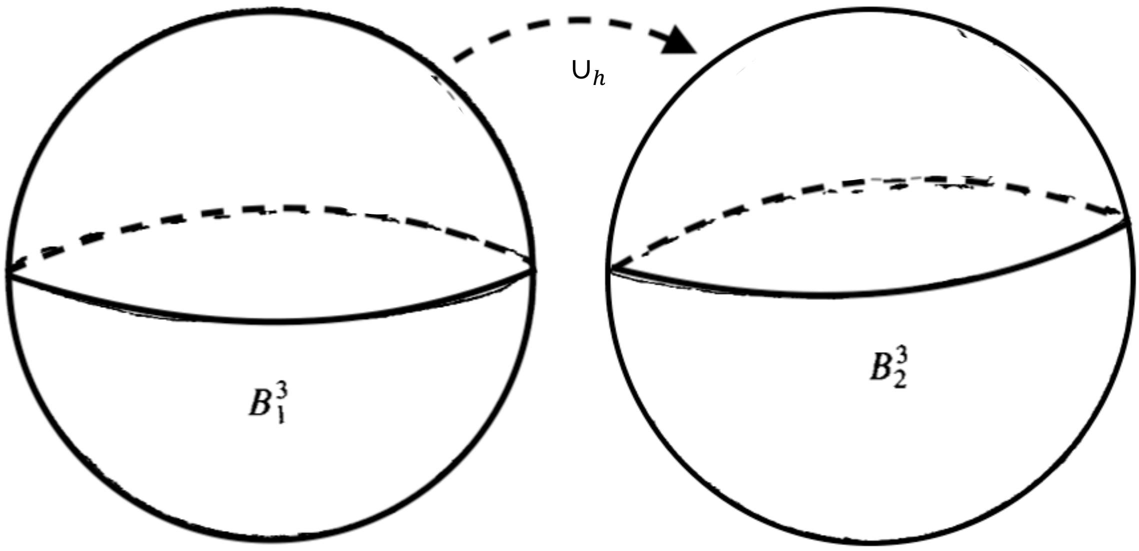

Figure 1. The 3-Sphere $\mathbb{S}^3$ as a union of two 3-Balls by a homeomorphism h. B_1^3 and B_2^3 designate the two 3-Balls. If a point p is contained in the interior of B_i^3, where i = 1 or 2, then any open set containing p is a neighborhood of p in $B_1^3 \cup_h B_2^3$.

There is another way to represent the 3-Sphere $\mathbb{S}^3$, which is as the one-point compactification of R^3. This is illustrated in Fig. 2.

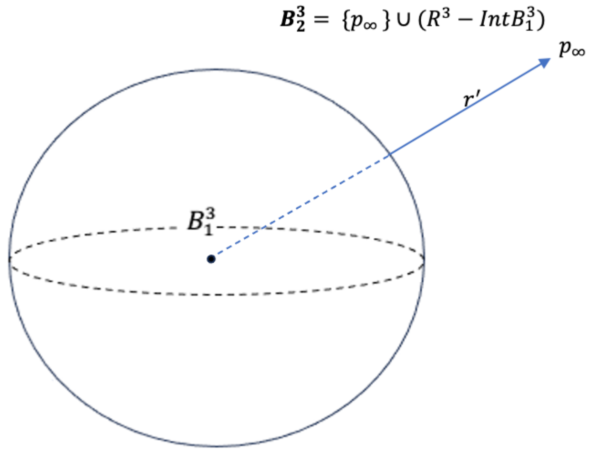

Figure 2. The 3-Sphere $\mathbb{S}^3$ as the one-point compactification of R^3. The point at infinity is designated as p_∞. B_1^3 is a ball in R^3 with the radius designated by the dashed line. The ball B_2^3 has its center at p_∞ and is given by $\{p_\infty\} \cup (R^3 - IntB_1^3)$, where *Int* means the interior. This means that the boundary of B_1^3 is also the boundary of B_2^3 (compare with Fig. 1). The radius of B_2^3 is $\{p_\infty\} \cup r'$.

If p is a point on the boundary of B_1^3, since the boundary of B_1^3 is also the boundary of B_2^3 the neighborhood of p (an open set containing p) is both in B_1^3 and B_2^3; i.e., a neighborhood of p in $B_1^3 \cup_h B_2^3$.

Additional insight into the 3-Sphere $\mathbb{S}^3$ can be gotten from the Hopf fibration (or Hopf map), which can be viewed as a map h of $\mathbb{S}^3$ onto $\mathbb{S}^2$ such that each point on $\mathbb{S}^2$ is mapped by h^{-1} to a distinct great circle on $\mathbb{S}^3$. Note that this mapping h is not a homeomorphism as it was above. This is shown in Fig. 3. The Hopf mapping can be considered to be the projection of the Hopf bundle, the locally trivial fiber space whose total space $\mathbb{S}^3$ has a base space $\mathbb{S}^2$ and fiber $\mathbb{S}^1$. Note, however, that the sphere $\mathbb{S}^3$ is not homeomorphic to the direct product $\mathbb{S}^2 \times \mathbb{S}^1$ since the fundamental groups of these spaces are not isomorphic.

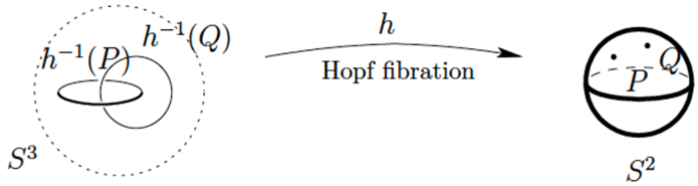

Figure 3. The Hopf fibration h mapping two linked great circles on $\mathbb{S}^3$ onto two points, P and Q, on the 2-Sphere $\mathbb{S}^2$. This mapping is not a homeomorphism. The inverse Hopf fibration, h^{-1}, maps P and Q into the linked great circles on $\mathbb{S}^3$. [Adapted from D.W. Lyons, *An Elementary Introduction to the Hopf Fibration*]

Perhaps counterintuitively, this shows that the great circles of $\mathbb{S}^3$ are linked, which can be seen from some of the models of $\mathbb{S}^3$.

Separation of the Matter and Antimatter Universes from $\mathbb{S}^3$

What is proposed here is the separation of the early $\mathbb{S}^3$ universe into a matter and separate antimatter universe. This would occur occurs at a time ~10^{-6}s after the big bang at the end of the quark-baryon transition period and before quark confinement and baryon-antibaryon annihilation occurs in the conventional theory. The time evolution in the matter and antimatter universes, while being opposite, continue to follow the usual laws of physics. For example, particle and antiparticle pairs could still be created and the Feynman Stückelberg interpretation of an antiparticle as a particle moving backwards in time would still hold.

This proposed separation of the universe into a matter and antimatter universe, just after the quark-baryon transition at ~10^{-6}s after the big bang, preserves the *CPT* theorem and avoids the problems associated with the idea of breaking the baryon number symmetry.

Boyle, Finn, and Turok[2] have also proposed a separation of the universe into a matter and an antimatter universe. They propose that the universe immediately after the big bang became a universe/anti-universe pair that directly emerges into the radiation-dominated era. They reason that during the radiation-dominated era the metric is $g_{\mu\nu} \propto \tau^2 \eta_{\mu\nu}$ where $\eta_{\mu\nu}$ is the Minkowski metric and τ is the conformal time. They then note that the metric has time reversal symmetry under $\tau \to -\tau$. This is the basis for their proposal that the universe is a universe/anti-universe pair that directly emerges after the big bang. They also couple their theory with the Feynman-Stückelberg interpretation of antiparticles being particles moving backwards in time.[3] This is very different from the proposal being made here that the division into a matter and antimatter universes is based on topological change at a time ~10^{-6}s after the big bang at the end of the quark-baryon transition period.

Figure 1 showed how the 3-Sphere can be illustrated as the union of two 3-Balls. Now consider the reverse process where the inverse of the homeomorphism h is applied to the 3-Sphere. This is shown in Fig. 4. As an aside, it might be of interest that the 3-Sphere $\mathbb{S}^3$ is the boundary of a closed 4-Ball B^4.

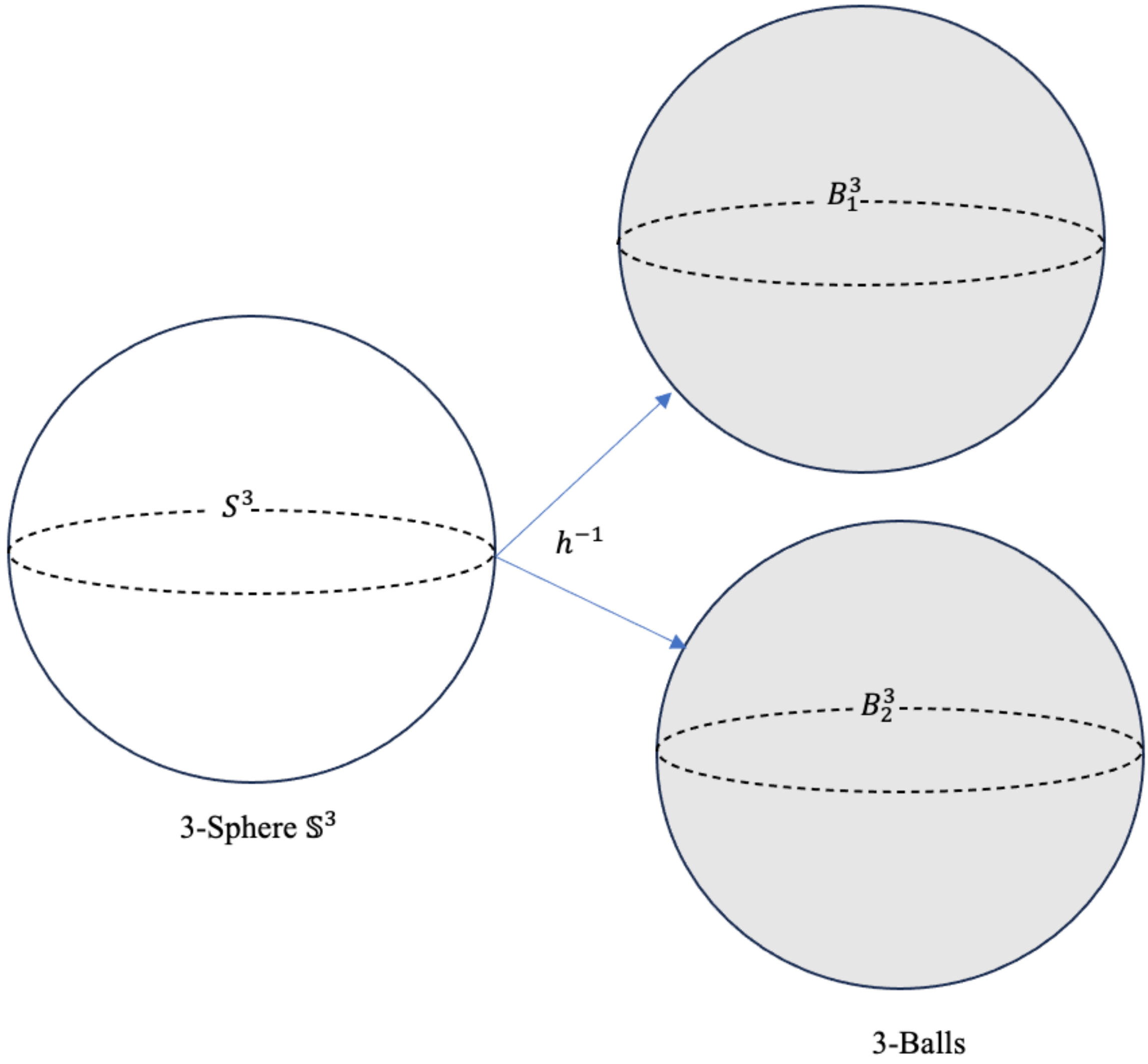

Figure 4. A 3-Sphere $\mathbb{S}^3$ becomes two 3-Balls under the inverse of the homeomorphism h used in Fig. 1. One dimension of the 3-Sphere is suppressed. The 3-Balls are shaded to show that the interior points of the balls are included.

The Separation of Baryons and Antibaryons

The key questions for the proposal being made here are: Is topological change consistent with general relativity; and, how does the separation of quarks and antiquarks occur?

With regard to topological change, Geroch[4] showed that for any spacetime containing *two separate* spacelike hypersurfaces of different topology there would have to be either closed timelike curves or singularities. In addition, Tipler[5] later showed that topology changing spacetimes are singular. In 1991, however, Horowitz[6] noted that the theorems of Geroch and Tipler basically showed that if a metric satisfies the Einstein field equation on a topologically changing manifold it cannot be well defined everywhere. He showed that the points at which the metric is ill defined do not have strong curvature singularities. To put it in his words, "The singularities can be very mild. In fact, they can be so mild that in some sense they are not there at all!" In particular, the curvature scalars are all finite and the curvature on the manifold does not diverge as one approaches the degenerate points.

Without invoking quantum gravity, Borde[7] found a way that classical general relativity can allow topological change and that is to eliminate the requirement that the foliation of spacetime into a series of spacelike hypersurfaces continues to exist as the topological change occurs.[†] He points out that there exit solutions to the Einstein field equations that do not admit a foliation of spacetime into spacelike hypersurfaces everywhere.

The net result from this literature is that there is no compelling reason to exclude the type of topological change shown in Fig. 4.

The quark-baryon transition occurs in some three phases[8] related to a critical temperature T_H, known as the Hagedorn temperature, corresponding to ~150 MeV per particle or a temperature exceeding 1.6×10^{12} °K. For T slightly greater than T_H forces between the components of the quark-gluon plasma continue to play some role, but for $T >> T_H$, the quark-gluon plasma,[9] consisting of quarks, antiquarks, and gluons, behaves as an ideal gas where the components of

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[†] Note the comments in endnote 3.

the quark-gluon plasma are free from interactions and move independently. This is the period during which the proposed topological change occurs.

The issue of the mechanism for the separation of quarks and antiquarks into B_1^3 and B_2^3, just after the quark-hadron transition at ~10^{-6}s after the big bang, remains to be explained.

Consider the Feynman-Stückelberg conception of antiparticles. Let $x(t)$ be the path of a particle in Minkowski space so that $dx^\mu = (dt, d\boldsymbol{x}(t))$. Define the Lorentz-invariant quantity $d\tau^2 = dx^\mu(t)dx_\mu(t)$. Then $d\tau(t)$ would represent a "length" along the path. It is given by $d\tau = \pm\sqrt{dx^\mu(t)dx_\mu(t)}$. For $d\boldsymbol{x} = 0$, the result is $d\tau = \pm dt$. Feynman and Stückelberg interpreted this to mean that for a particle moving along $d\tau = -dt$, in the opposite sense to the dt in our Lorentz frame, is an antiparticle. Thus, the Feynman-Stückelberg conception of antiparticles arises in classical physics so that one has "classical antiparticles".[10] $d\tau = +dt$ corresponds to the forward light cone and $d\tau = -dt$ to the backward light cone. This is shown in Fig. 5.

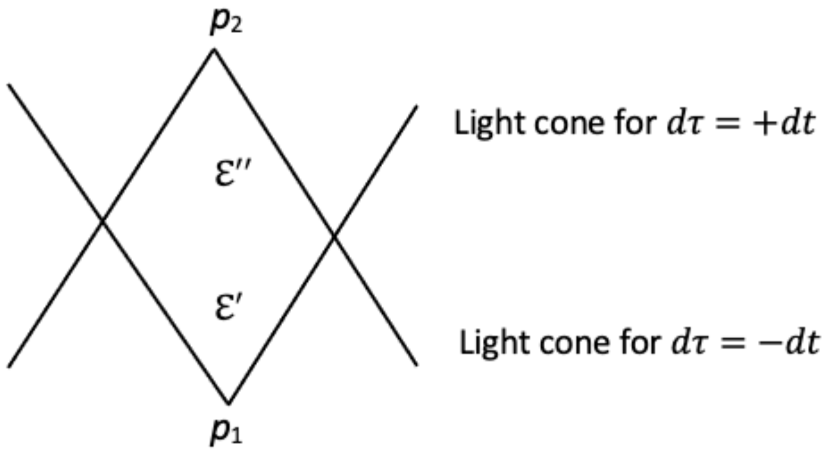

Figure 5. The light cones corresponding to $d\tau = +dt$ and $d\tau = -dt$ at the points p_1 and p_2. $\mathcal{E}'$ and $\mathcal{E}''$ are two events. Both light cones correspond to legitimate Lorentz frames where time evolves in opposite directions.

In the Lorentz frame corresponding to $d\tau = +dt$, event $\mathcal{E}'$ occurs before $\mathcal{E}''$ and in the frame corresponding to $d\tau = -dt$ it occurs after $\mathcal{E}''$. What this shows is that both solutions to $d\tau = \pm\sqrt{dx^\mu(t)dx_\mu(t)}$ are equally valid definitions for the passage of proper time. This is what convinced Feynman and Stückelberg that a particle evolving in either Lorentz frame viewed from the other frame is seen as an antiparticle.

Return now to Fig. 4. Just after quark-hadron transition at ~10^{-6}s after the big bang is when the topological change shown in Fig. 4 is assumed to occur. These two 3-Balls would represent two separate universes, one containing matter (corresponding to $\tau = +dt$) and the other antimatter (corresponding to $\tau = -dt$). At the moment of topological change, there would be a separation of particles and antiparticles into the 3-Ball whose time flow matches the particle or antiparticle. A particle moving forward in time in one 3-Ball would be viewed from the other 3-Ball as being an antiparticle. In this way the mixture of matter and antimatter in $\mathbb{S}^3$ would be separated by the proposed topological change into two universes one having matter and the other antimatter; and thereby saving CPT symmetry.

Note, however, that these universes would have a boundary unlike the 3-Sphere they came from. But from the first of Eqs. (1) this boundary would not be visible in the later universe as $S(t)$ caused its expansion.

The proposal above would preserve the CPT theorem, and it is important that CPT symmetry not be violated since it is this symmetry that implies that every particle has an antiparticle, that the mass of the particle and antiparticle is equal, and if the particle is unstable its lifetime is the same as its antiparticle. If either C, P, or T is violated there will be a violation of one of the other two. It has also been shown that if CPT invariance is violated in an interacting quantum field theory, then that theory also violates Lorentz invariance.[11]

Some Philosophical Comments

The theory given above is obviously quite speculative. Two separate universes are postulated, which both come into existence at the moment of topological change when particles and antiparticles are separated into two 3-Balls each having a time direction opposite to the other. Since the two universes, matter and antimatter, are disjoint and have no contact whatsoever, from a philosophical point of view, the theory is not falsifiable.

The role of falsifiability is discussed extensively in Karl Popper's book *The Logic of Scientific Discovery*. In particular, if a theory is not falsifiable this does not mean it is wrong. As Popper put it, "Falsifiability separates two kinds of perfectly meaningful statements: the falsifiable and the non-falsifiable". He uses the concept of falsifiability as a "criterion for deciding whether or not a theoretical system belongs to empirical science".

With regard to their geometrical relationship, one cannot define a three-dimensional distance between the two universes and they could well be in the same place in 3-space since the distance between them is time-like. As a result, it would not be possible to make any experimental observations in one of the 3-Ball universes that could confirm the existence of the other. Thus, by Popper's criterion, the theory given here does not belong to empirical science. A couple of examples of other non-empirical theories that are currently being explored are string theory and multi-universe theory.

REFERENCES AND COMMENTS

[1] R. H. Bing, *The Geometric Topology of 3-Manifolds* (The American Mathematical Society, Rhode Island 1983), Colloquium Publications, v. 40. Homeomorphic spaces are "the same" from a topological point of view and satisfy the conditions that it is a bijection (one-to-one and onto), it is continuous, and the inverse function is continuous; i.e., an open mapping.

[2] L. Boyle, K. Finn, and N. Turok, “CPT-Symmetric Universe”, *Phys. Rev. Lett.* **121** (2018), p. 251301. See also their longer paper available at arXiv:1803.08930.

[3] In the general view of time, moving backwards in time would take one to a three-dimensional space as it was in the past with the configuration of matter being what it was at each instant of time in the past. In this conception of time, three-dimensional hypersurfaces continue to exist in the sense that moving backward in time recapitulates three-dimensional space exactly as it was in the past. However, this usual conception of time imposes our own psychological time on the past, it does not mean that the physical past actually continues to exist. The Feynman-Stückelberg conception of antiparticles is fully consistent with the non-existence of past three-dimensional hypersurfaces.

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[5] F. Tipler, *Ann. Phys*. **108**, 1 (1977).

[6] G. T. Horowitz, “Topological Change in General Relativity”, arXiv:hep-th/9109130. (To appear in the proceedings of the Sixth Marcel Grossman Meeting held in Kyoto, Japan, June 24-29, 1991).

[7] A. Borde, “Topological Change in Classical General Relativity”, arXiv:gr-qc/9406053v1 (1994). See also, *Bull. Astr. Soc. India* **25**, 571 (1997).

[8] S. Bonomett and O. Pantano, “Cosmological aspects of the quark-hadron transition”, *Astron. Astrophys*, **130**, 49-52 (1984).

[9] J. Letessier and J. Rafelski, *Hadrons and Quark-Gluon Plasma* (Cambridge University Press, Cambridge 2022).

[10] J. P. Costella,et al., *Am. J. Phys.* **65**, 835-841 (1997).

[11] O. W. Greenberg, Phys. Rev. Lett. **89**, 231602 (2002).